Complete and ready simulation semantics are not finitely based over BCCSP, even with a singleton alphabet☆

Luca Aceto[a,*], David de Frutos Escrig[b], Carlos Gregorio-Rodríguez[b], Anna Ingolfsdottir[a]

[a] ICE-TCS, School of Computer Science, Reykjavik University, Menntavegur 1, IS 101 Reykjavik, Iceland
[b] Departamento de Sistemas Informáticos y Computación, Universidad Complutense de Madrid, Spain

Abstract

This note shows that the complete and the ready simulation preorders do not have a finite inequational basis over the language BCCSP when the set of actions is a singleton. Moreover, the equivalences induced by those preorders do not have a finite (in)equational axiomatization either. These results are in contrast with a claim of finite axiomatizability for those semantics in the literature, which was based on the erroneous assumption that they coincide with complete trace semantics in the presence of a singleton set of actions.

Keywords: Concurrency, process algebra, complete simulation, ready simulation, equational logic, non-finitely based algebras

1. Introduction

Since the seminal work by Hennessy and Milner, eventually published in [12], the search for (in)equational axiomatizations of notions of behavioural semantics for fragments of process algebras has been one of the classic topics of investigation within concurrency theory. A complete axiomatization of a behavioural semantics yields a purely syntactic and model-independent characterization of the semantics of the process algebra, and paves the way to the application of theorem-proving techniques in establishing whether two process descriptions exhibit related behaviours.

An (in)equational axiomatization is *ground complete* if it can prove all the valid (in)equivalences relating terms with no occurrences of variables in the process algebra of interest. It is *complete* when it can be used to derive all the valid (in)equivalences. A complete axiom system is also referred to as a *basis* for the algebra it axiomatizes.

A review of ground-complete equational axiomatizations for many of the behavioural semantics in van Glabbeek's linear time-branching time spectrum is offered in [11]. The equational axiomatizations offered ibidem are over the language BCCSP, a common fragment of Milner's CCS [15] and Hoare's CSP [13] suitable for describing finite synchronization trees, and characterize the differences between behavioural semantics in terms of a few revealing axioms. In [10], two of the authors of this paper presented a unification of the axiomatizations of all the semantics in the linear time-branching time spectrum. This unification is achieved by means of conditional axioms that provide a simple and clear picture of the similarities and differences between all the semantics.

The article [3] surveys results on the existence of finite, complete equational axiomatizations of behavioural equivalences over fragments of process algebras up to 2005. Some of the results on the (non)existence of finite, complete (in)equational axiomatizations of behavioural semantics over process algebras that have been obtained since the publication of that survey may be found in [1, 2, 4,

☆Luca Aceto and Anna Ingolfsdottir have been partially supported by the projects 'New Developments in Operational Semantics' (nr. 080039021) and 'Meta-theory of Algebraic Process Theories' (nr. 100014021) of the Icelandic Fund for Research. The work on the paper was carried out while Luca Aceto held an Abel Extraordinary Chair at Universidad Complutense de Madrid, Spain, supported by the NILS Mobility Project.

*Corresponding author

Email addresses: luca@ru.is (Luca Aceto), defrutos@sip.ucm.es (David de Frutos Escrig), cgr@sip.ucm.es (Carlos Gregorio-Rodríguez), annai@ru.is (Anna Ingolfsdottir)

5, 6, 8].

In the setting of BCCSP, in a veritable tour de force that collects and unifies the results in a series of conference articles, Chen, Fokkink, Luttik and Nain have offered in [9] a definitive classification of the status of the finite basis problem—that is, the problem of determining whether a behavioural equivalence has a finite, complete, equational axiomatization over the chosen process algebra—for all the semantics in van Glabbeek's spectrum. Table 1 on page 517 of that article summarizes their findings for each semantics in the spectrum. Notable later results by Chen and Fokkink presented in [8] give the first example of a semantics—the so-called *impossible future semantics* from [19]—where the preorder defining the semantics can be finitely axiomatized over BCCSP, but its induced equivalence cannot.

It is remarkable that the finite axiomatizability of several notions of semantics depends crucially on the cardinality of the set of actions processes may perform. Moreover, different notions of semantics exhibit different axiomatizability properties even when the set of actions is a singleton. Table 1 on page 517 of [9] indicates that complete and ready simulation equivalence [7, 11, 14] afford finite, complete equational axiomatizations in case the set of actions is a singleton. (Note that complete and ready simulation equivalence/preorder coincide when the set of actions is a singleton.) However, that paper offers no proof for those claims. The authors simply write on page 494 of their paper that:

> If $|A| = 1$, then all semantics from completed traces up to ready simulation coincide with completed trace semantics, while simulation coincides with trace semantics. And there exists a finite basis for the equational theories of BCCSP modulo completed trace and trace equivalence if $|A| = 1$.

However, complete and ready simulation semantics are finer than complete trace semantics, even in the presence of a single action. For example, the processes described by the terms $a(a\mathbf{0} + aa\mathbf{0})$ and $aa\mathbf{0} + aaa\mathbf{0}$ afford the same complete traces, but the former cannot be simulated by the latter in complete trace semantics.

Our aim in this paper is to prove that, contrary to the information presented in Table 1 of [9], complete and ready simulation equivalence/preorder do *not* have a finite (in)equational basis over BCCSP when the set of actions is a singleton. In fact, our results entail that the collection of valid (in)equations in at most one variable does *not* have an (in)equational basis consisting of sound inequations whose 'depth' is bounded. This result completes the classification of the finite basis problem over BCCSP with a singleton action set for the behavioural semantics in van Glabbeek's linear time-branching time spectrum. Indeed, when the set of actions is a singleton, simulation and trace semantics coincide and all the semantics between possible worlds and completed traces collapse.

The paper is organized as follows. Section 2 is devoted to preliminaries on BCCSP, complete and ready simulation semantics, and equational logic. Section 3 is entirely devoted to the proof of our nonfinite axiomatizability results. Section 4 offers some concluding remarks.

2. Preliminaries

Syntax of BCCSP. We work with BCCSP [11, 13, 15] over the singleton action set $A = \{a\}$. This language is a basic process algebra for expressing finite process behaviour. Its syntax consists of closed (process) terms p, q that are constructed from a constant $\mathbf{0}$, a binary operator $\_ + \_$ called *alternative composition*, and the unary *prefix* operator $a\_$. Open terms t, u can, moreover, contain occurrences of variables from a countably infinite set V (with typical elements x, y, z). The *depth* of a term t is the largest nesting of prefix operators in t.

In what follows, for each $n \geq 0$, we use $a^n t$ to stand for the term t if $n = 0$, and for $a(a^{n-1}t)$ if $n > 0$.

A (closed) substitution maps variables in V to (closed) terms. For every term t and substitution σ, the term $\sigma(t)$ is obtained by replacing every occurrence of a variable x in t by $\sigma(x)$. Note that $\sigma(t)$ is closed if σ is a closed substitution.

Transition rules. Intuitively, closed BCCSP terms represent finite process behaviours, where $\mathbf{0}$ does not exhibit any behaviour, $p + q$ is the nondeterministic choice between the behaviours of p and q, and ap executes action a to transform into p. This intuition is captured, in the style of Plotkin [18], by the transition rules below, which give rise to a-labelled transitions between closed terms.

$$\frac{}{ax \xrightarrow{a} x} \qquad \frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'}$$

The operational semantics is extended to open terms by assuming that variables do not exhibit any behaviour.

For each BCCSP term t, we define

$$I(t) = \{a \mid t \xrightarrow{a} t' \text{ for some } t'\}.$$

Note that, since we have only one action, $I(t)$ is either $\emptyset$ or $\{a\}$.

Complete and ready simulation. We define the following three variations on the notion of simulation over closed BCCSP terms.

Definition 2.1 (Simulations). *A binary relation $\mathcal{R}$ over closed BCCSP terms is:*

- *a* simulation *if $p \mathcal{R} q$ and $p \xrightarrow{a} p'$ imply $q \xrightarrow{a} q'$ for some q' with $p' \mathcal{R} q'$;*
- *a* complete simulation *if it is a simulation that satisfies the following condition:*

  $$p \mathcal{R} q \text{ and } I(p) = \emptyset \text{ imply } I(q) = \emptyset;$$

- *a* ready simulation *if it is a simulation that satisfies the following condition:*

  $$p \mathcal{R} q \text{ implies } I(q) \subseteq I(p).$$

We write $p \precsim_{CS} q$ if there is a complete simulation $\mathcal{R}$ with $p \mathcal{R} q$, and $p \precsim_{RS} q$ if there is a ready simulation $\mathcal{R}$ with $p \mathcal{R} q$. We will refer to $\precsim_{CS}$ as the complete simulation preorder*, and to $\precsim_{RS}$ as the* ready simulation preorder*.*

We say that closed BCCSP terms p, q are complete simulation equivalent *iff p and q are related by the kernel of $\precsim_{CS}$, that is when both $p \precsim_{CS} q$ and $q \precsim_{CS} p$ hold. Ready simulation equivalence is the kernel of $\precsim_{RS}$.*

Let $\precsim \in \{\precsim_{CS}, \precsim_{RS}\}$. We define $t \precsim u$ if $\sigma(t) \precsim \sigma(u)$ for each closed substitution σ.

It is well known that $\precsim_{RS}$ is included in $\precsim_{CS}$ over arbitrary labelled transition systems, and that the converse inclusion fails in general—see, e.g., [11]. In our setting, however, the complete simulation preorder and the ready simulation preorder coincide, because the complete simulation preorder is a ready simulation. This follows because if $p \precsim_{CS} q$ then either $I(p) = I(q) = \emptyset$ or $I(p) = I(q) = \{a\}$. Therefore, in what follows, we will focus on proving our results for complete simulation semantics.

Inequational logic. An *inequation* (respectively, an *equation*) over the language BCCSP is a formula of the form $t \leq u$ (respectively, $t = u$), where t and u are BCCSP terms. An *(in)equational axiom system* is a collection of (in)equations over the language BCCSP. An equation $t = u$ is derivable from an equational axiom system E if it can be proven from the axioms in E using the rules of equational logic (viz. reflexivity, symmetry, transitivity, substitution and closure under BCCSP contexts).

$$\frac{}{t = t} \quad \frac{t = u}{u = t} \quad \frac{t = u \quad u = v}{t = v}$$

$$\frac{t = u}{\sigma(t) = \sigma(u)} \quad \frac{t = u}{at = au} \quad \frac{t = u \quad t' = u'}{t + t' = u + u'}$$

For the derivation of an inequation $t \leq u$ from an inequational axiom system E, the rule for symmetry is omitted.

It is well known that, without loss of generality, one may assume that substitutions happen first in (in)equational proofs, i.e., that the rule

$$\frac{t = u}{\sigma(t) = \sigma(u)}$$

may only be used when its premise is one of the (in)equations in E. Moreover, by postulating that for each equation in E also its symmetric counterpart is present in E, one may assume that applications of symmetry happen first in equational proofs, i.e., that the rule

$$\frac{t = u}{u = t}$$

is never used in equational proofs. (See, e.g., [9, page 497] for a thorough discussion of this 'normalized equational proofs'.) In the remainder of this paper, we shall always tacitly assume that equational axiom systems are closed with respect to symmetry. Note that, with this assumption, there is no difference between the rules of inference of equational and inequational logic. In what follows, we shall consider an equation $t = u$ as a shorthand for the pair of inequations $t \leq u$ and $u \leq t$.

The depth of $t \leq u$ and $t = u$ is the maximum of the depths of t and u. The depth of a collection of (in)equations is the supremum of the depths of its elements.

An inequation $t \leq u$ is *sound* with respect to $\precsim_{CS}$ if $t \precsim_{CS} u$ holds. For example, as our readers can readily check, the inequation

$$ax \leq ax + y \qquad (1)$$

is sound with respect to $\precsim_{CS}$. An (in)equational axiom system E is sound with respect to $\precsim_{CS}$ if so is each (in)equation in E.

The core axioms A1–A4 for BCCSP given below are classic and stem from [12]. They are ω-complete [16], and sound and ground-complete [12, 15], over BCCSP (over any nonempty set of actions) modulo bisimulation equivalence [15, 17], which is the finest semantics in van Glabbeek's spectrum [11].

$$\begin{array}{llcl} \text{A1} & x + y & = & y + x \\ \text{A2} & (x+y)+z & = & x+(y+z) \\ \text{A3} & x + x & = & x \\ \text{A4} & x + \mathbf{0} & = & x \end{array}$$

In what follows, for notational convenience, we consider terms up to the least congruence generated by axioms A1–A4, that is, up to bisimulation equivalence.

3. The negative result

Our aim in what follows is to show the following theorem.

Theorem 3.1. *If $A = \{a\}$ then the (in)equational theory of $\precsim_{CS}$ over BCCSP does not have a finite (in)equational basis. In particular, the following statements hold true.*

1. *No finite set of sound inequations over BCCSP modulo $\precsim_{CS}$ can prove all of the sound inequations in the family*
$$a^n x \leq a^n \mathbf{0} + a^n(x + a\mathbf{0}) \quad (n \geq 1).$$
2. *No finite set of sound (in)equations over BCCSP modulo $\precsim_{CS}$ can prove all of the sound equations in the family*
$$a^n x + a^n \mathbf{0} + a^n(x + a\mathbf{0}) = a^n \mathbf{0} + a^n(x + a\mathbf{0}),$$
with $n \geq 1$.

Theorem 3.1 is a corollary of the following result.

Theorem 3.2. *Assume that $A = \{a\}$. Let E be a collection of inequations whose elements are sound modulo $\precsim_{CS}$ and have depth smaller than n. Suppose furthermore that the inequation $t \leq u$ is derivable from E and that $u \precsim_{CS} a^n \mathbf{0} + a^n(x + a\mathbf{0})$. Then $t \xrightarrow{a^n} x$ implies $u \xrightarrow{a^n} x$.*

Having shown the above result, statement 1 in Theorem 3.1 can be proved as follows. Let E be a finite inequational axiom system that is sound modulo $\precsim_{CS}$. Pick n larger than the depth of E. (Such an n exists since E is finite.) Then, by Theorem 3.2, E cannot prove the valid inequation

$$a^n x \leq a^n \mathbf{0} + a^n(x + a\mathbf{0}),$$

and is therefore incomplete. Indeed,

$$a^n x \xrightarrow{a^n} x.$$

On the other hand, the only terms t such that

$$a^n \mathbf{0} + a^n(x + a\mathbf{0}) \xrightarrow{a^n} t$$

holds are $\mathbf{0}$ and $x + a\mathbf{0}$. So $a^n \mathbf{0} + a^n(x + a\mathbf{0}) \xrightarrow{a^n} x$ does not hold.

Statement 2 in Theorem 3.1 is a corollary of Theorem 3.1(1). To see this, assume Theorem 3.1(1) and suppose, towards a contradiction, that there is a finite set of sound (in)equations over BCCSP modulo $\precsim_{CS}$ that can prove all of the equations in the family

$$a^n x + a^n \mathbf{0} + a^n(x + a\mathbf{0}) = a^n \mathbf{0} + a^n(x + a\mathbf{0}),$$

with $n \geq 1$. Recall that we may assume that E is closed with respect to symmetry and that, under this assumption, there is no difference between the rules of inference of equational and inequational logic. Thus E can prove all the inequations

$$a^n x + a^n \mathbf{0} + a^n(x + a\mathbf{0}) \leq a^n \mathbf{0} + a^n(x + a\mathbf{0}),$$

with $n \geq 1$. Observe now that the sound inequation (1) can be used to show that

$$a^n x \leq a^n x + a^n \mathbf{0} + a^n(x + a\mathbf{0}) \quad (n \geq 1).$$

Therefore, by transitivity, the finite set of sound inequations $E \cup \{(1)\}$ can prove all of the inequations in the family

$$a^n x \leq a^n \mathbf{0} + a^n(x + a\mathbf{0}) \quad (n \geq 1).$$

This, however, contradicts Theorem 3.1(1).

In the remainder of this note, we shall present a proof of Theorem 3.2. In order to show that result, we shall first prove that the property mentioned in that statement holds true for instantiations of sound inequations whose depth is smaller than n.

Next we use this fact to argue that the stated property is preserved by arbitrary inequational derivations from a finite collection of inequations whose elements have depth smaller than n and are sound modulo $\precsim_{CS}$.

Definition 3.3. *We say that a term t has an occurrence of variable x at depth k if there is some term t' such that $t \xrightarrow{a^k} x + t'$.*

For example, $ax + a\mathbf{0}$ has an occurrence of x at depth 1 because $ax + a\mathbf{0} \xrightarrow{a} x$ and $x = x + \mathbf{0}$.

Lemma 3.4. *Assume that $t \precsim_{CS} u$ and that t has an occurrence of variable x at depth k. Then u has an occurrence of variable x at depth k.*

Proof Assume that $t \precsim_{CS} u$ and that t has an occurrence of variable x at depth k. Let m be larger than the depth of u. Consider the closed substitution σ mapping x to $a^m\mathbf{0}$ and every other variable to $\mathbf{0}$. Since t has an occurrence of variable x at depth k, it is easy to see that $\sigma(t) \xrightarrow{a^{k+m}} \mathbf{0}$. As $\sigma(t) \precsim_{CS} \sigma(u)$ because $t \precsim_{CS} u$ by assumption, it must be the case that $\sigma(u) \xrightarrow{a^{k+m}} p$ for some p such that $\mathbf{0} \precsim_{CS} p$. Such a p must be $\mathbf{0}$. As the depth of u is smaller than m, σ maps all variables different from x to $\mathbf{0}$, $\sigma(u) \xrightarrow{a^{k+m}} p$ and p is $\mathbf{0}$, it follows that $u \xrightarrow{a^k} x + u'$ for some u', and we are done. □

The following lemma is the first stepping stone towards the proof of Theorem 3.2. It establishes that the property mentioned in that statement holds true for instantiations of sound inequations whose depth is smaller than n.

Lemma 3.5. *Suppose that $t \precsim_{CS} u$ and that n is larger than the depth of t. Then $\sigma(t) \xrightarrow{a^n} x$ implies $\sigma(u) \xrightarrow{a^n} x$.*

Proof Assume that $\sigma(t) \xrightarrow{a^n} x$. Since n is larger than the depth of t, there are some $0 \leq i < n$ and some variable z such that t has an occurrence of variable z at depth i and $\sigma(z) \xrightarrow{a^{n-i}} x$. As $t \precsim_{CS} u$, Lemma 3.4 yields that u has an occurrence of variable z at depth i. Therefore $\sigma(u) \xrightarrow{a^n} x$, which was to be shown. □

We will now argue that the property stated in Theorem 3.2 is preserved by arbitrary inequational derivations from a finite collection of inequations whose elements are sound modulo $\precsim_{CS}$ and have depth smaller than n. The following lemma will allow us to handle closure under action prefixing in that proof.

Lemma 3.6. *Assume that $at \precsim_{CS} au \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0})$, and that $at \xrightarrow{a^n} x$. Then $au \xrightarrow{a^n} x$.*

Proof Assume that

$$at \precsim_{CS} au \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0}),$$

and that $at \xrightarrow{a^n} x$. Since at has an occurrence of x at depth n, by Lemma 3.4 so does au. This means that $au \xrightarrow{a^n} x + u'$ for some u'. Observe now that $au \xrightarrow{a^n} \mathbf{0}$ cannot hold, because this would contradict $au \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0})$. Indeed, assume, towards a contradiction, that $au \xrightarrow{a^n} \mathbf{0}$. Consider a closed substitution that maps x to $a\mathbf{0}$. Then $\sigma(au) \xrightarrow{a} \sigma(u)$. The only terms that can be reached from $\sigma(a^n\mathbf{0} + a^n(x + a\mathbf{0}))$ via $\xrightarrow{a}$ are $a^{n-1}\mathbf{0}$ and $a^{n-1}(a\mathbf{0} + a\mathbf{0})$. However, neither $\sigma(u) \precsim_{CS} a^{n-1}\mathbf{0}$ nor $\sigma(u) \precsim_{CS} a^{n-1}(a\mathbf{0} + a\mathbf{0})$ holds. Indeed, the former fails because

$$\sigma(u) \xrightarrow{a^{n-1}} a\mathbf{0} + \sigma(u') \not\precsim_{CS} \mathbf{0},$$

and the latter because

$$\sigma(u) \xrightarrow{a^{n-1}} \mathbf{0} \not\precsim_{CS} a\mathbf{0} + a\mathbf{0}.$$

Consider now the closed substitution $\sigma_{\mathbf{0}}$ that maps all variables to $\mathbf{0}$. Then $\sigma_{\mathbf{0}}(at) \xrightarrow{a^n} \mathbf{0}$ because $at \xrightarrow{a^n} x$ by the proviso of the lemma. As $at \precsim_{CS} au$, we have that $\sigma_{\mathbf{0}}(at) \precsim_{CS} \sigma_{\mathbf{0}}(au)$. Therefore, $\sigma_{\mathbf{0}}(au) \xrightarrow{a^n} \mathbf{0}$. Since, by our earlier observation, $au \xrightarrow{a^n} \mathbf{0}$ cannot hold, we have that $au \xrightarrow{a^n} u''$ for some u'' such that $u'' \neq \mathbf{0}$ and $\sigma_{\mathbf{0}}(u'') = \mathbf{0}$. Such a u'' can only contain occurrences of the variable x (by Lemma 3.4 and the assumption that $au \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0})$). Therefore $u'' = x$ and we are done. □

We now have all the necessary ingredients to complete the proof of Theorem 3.2, and therefore of statement 1 in Theorem 3.1.

Proof *(of Theorem 3.2)* Assume that E is a collection of inequations whose elements are sound modulo $\precsim_{CS}$ and have depth smaller than n. Suppose furthermore that

- the inequation $t \leq u$ is derivable from E,
- $u \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0})$, and
- $t \xrightarrow{a^n} x$.

(Observe that n is positive because it is larger than the depth of E.) We shall prove that $u \xrightarrow{a^n} x$ by induction on a derivation of $t \leq u$ from E. We proceed by examining the last rule used in the proof of $t \leq u$ from E. The case of reflexivity is trivial and that of transitivity follows by applying the inductive hypothesis twice. If $t \leq u$ is proved by instantiating an inequation in E, then the claim follows by Lemma 3.5. We are therefore left with the congruence rules, which we examine separately below.

- Suppose that E proves $t \leq u$ because $t = at'$, $u = au'$ and E proves $t' \leq u'$ by a shorter inference. By the soundness of E and the proviso of the theorem, we have that

  $$t = at' \precsim_{CS} u = au' \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0})$$

  and $t \xrightarrow{a^n} x$. Lemma 3.6 now yields $u \xrightarrow{a^n} x$, as required.

- Suppose that E proves $t \leq u$ because $t = t_1 + t_2$, $u = u_1 + u_2$ and E proves $t_i \leq u_i$, $1 \leq i \leq 2$, by shorter inferences. Since $t \xrightarrow{a^n} x$ and n is positive, we may assume, without loss of generality, that $t_1 \xrightarrow{a^n} x$. Using the soundness of E and the fact that $I(t_1) = \{a\}$, it is not hard to see that

  $$u_1 \precsim_{CS} a^n\mathbf{0} + a^n(x + a\mathbf{0}).$$

  Therefore we may apply the induction hypothesis to infer that $u_1 \xrightarrow{a^n} x$. Hence, as n is positive, $u \xrightarrow{a^n} x$, as required.

This completes the proof. □

Corollary 3.7. *If $A = \{a\}$ then the collection of (in)equations in at most one variable that hold in complete (or ready) simulation semantics over BCCSP does not have a finite (in)equational basis. Moreover, for each n, the collection of all sound (in)equations of depth at most n cannot prove all the valid (in)equations in at most one variable that hold in complete (or ready) simulation semantics over BCCSP.*

4. Concluding remarks

It is instructive to compare the results we have presented in this note with those offered in [9]. In their proofs of the nonfinite axiomatizability results for complete and ready simulation equivalence (Sections 7 and 8 in [9]), Chen, Fokkink, Luttik and Nain assume that the set of actions contains at least two actions. (The proof for ready simulation equivalence also relies on the finiteness of the set of actions.) Their proofs are entirely correct. However, if one analyzes them carefully, one sees that the assumption that the set of actions contains at least two actions is only used in the proof of the negative result for complete simulation equivalence via a reference to Lemma 3(3) in that paper. In that statement, the assumption that the set of actions is of cardinality at least two is necessary because those authors prove the lemma for the trace preorder, which allows them to use it for all the other semantics as well. However, as Lemma 3.4 in this paper shows, Lemma 3(3) in [9] holds for complete simulation semantics (in fact, for complete trace semantics) even if A is a singleton. This means that, modulo this change, the proof by Chen, Fokkink, Luttik and Nain in [9, Section 7] shows that complete simulation equivalence has no finite equational basis even when A is a singleton.

In the proof of the nonfinite axiomatizability result for ready simulation equivalence, the assumption that A contains at least two actions is also used in the proof of Lemma 33 in [9]. However, that use can be overcome when A is a singleton by employing the weaker 'invariant property' stated in our Theorem 3.2.

The proof of our main result has been inspired by those in [9]. However, it uses an invariant based on the operational semantics (the ability to perform a sequence of n a-labelled transitions ending in x) rather than an 'equivalence-based one' (having a summand that is equivalent to $a^n x$), and a family of inequations in one variable rather than in two variables, as is the case for complete simulation equivalence in [9]. This leads to a simplification in the proofs (compare our Lemma 3 with Lemmas 29 and 33 in [9]) and to a stronger negative result: the collection of all valid (in)equations in at most one variable is not finitely axiomatizable.

The family of (in)equations we use is valid in complete simulation semantics for an arbitrary action set. So our proof can be used to show Theorem 32 in [9] in lieu of the one in that reference. One just needs to change the statement of Lemma 3.4 by replacing 'at depth k' with 'reachable via a sequence of actions s'.

The family of (in)equations we use in our proof is *not* valid in ready simulation semantics when the set of actions is not a singleton. However, even in that case, our proof strategy can be used to show that no collection of (in)equations of bounded depth that are valid in complete simulation semantics can prove all the equivalences in the family of equations on page 516 in [9], which are valid in ready simulation semantics. This proves Theorem 32 (when the set of actions is finite) and Theorem 36 in [9] in one fell swoop.

Acknowledgements. We thank the anonymous referees for their thorough reading of the paper and for spotting a few infelicities in the submitted paper.

References

[1] L. Aceto, S. Capobianco, A. Ingolfsdottir, and B. Luttik. The equational theory of prebisimilarity over basic CCS with divergence. *Inf. Process. Lett.*, 108(5):284–289, 2008.

[2] L. Aceto, T. Chen, W. Fokkink, and A. Ingolfsdottir. On the axiomatisability of priority. *Mathematical Structures in Computer Science*, 18(1):5–28, 2008.

[3] L. Aceto, W. Fokkink, A. Ingolfsdottir, and B. Luttik. Finite equational bases in process algebra: Results and open questions. In *Processes, Terms and Cycles*, volume 3838 of *Lecture Notes in Computer Science*, pages 338–367. Springer, 2005.

[4] L. Aceto, W. Fokkink, A. Ingolfsdottir, and B. Luttik. A finite equational base for CCS with left merge and communication merge. *ACM Trans. Comput. Log.*, 10(1), 2009.

[5] L. Aceto, W. Fokkink, A. Ingolfsdottir, and M. R. Mousavi. Lifting non-finite axiomatizability results to extensions of process algebras. *Acta Informatica*, 47(3):147–177, 2010.

[6] L. Aceto, A. Ingolfsdottir, B. Luttik, and P. van Tilburg. Finite equational bases for fragments of CCS with restriction and relabelling. In G. Ausiello, J. Karhumäki, G. Mauri, and C.-H. L. Ong, editors, *Fifth IFIP International Conference On Theoretical Computer Science - TCS 2008, IFIP 20th World Computer Congress, TC 1, Foundations of Computer Science, September 7-10, 2008, Milano, Italy*, volume 273 of *IFIP*, pages 317–332. Springer, 2008.

[7] B. Bloom, S. Istrail, and A. R. Meyer. Bisimulation can't be traced. *Journal of the ACM*, 42(1):232–268, 1995.

[8] T. Chen and W. Fokkink. On the axiomatizability of impossible futures: Preorder versus equivalence. In *LICS 2008*, pages 156–165. IEEE Computer Society, 2008.

[9] T. Chen, W. Fokkink, B. Luttik, and S. Nain. On finite alphabets and infinite bases. *Information and Computation*, 206(5):492–519, 2008.

[10] D. de Frutos-Escrig, C. Gregorio-Rodríguez, and M. Palomino. On the unification of process semantics: Equational semantics. *Electronic Notes in Theoretical Computer Science*, 249:243–267, 2009.

[11] R. van Glabbeek. The linear time – branching time spectrum I; the semantics of concrete, sequential processes. In J. Bergstra, A. Ponse, and S. Smolka, editors, *Handbook of Process Algebra*, chapter 1, pages 3–99. Elsevier, 2001. Available at `http://boole.stanford.edu/pub/spectrum1.ps.gz`.

[12] M. Hennessy and R. Milner. Algebraic laws for nondeterminism and concurrency. *Journal of the ACM*, 32:137–161, 1985.

[13] C. Hoare. *Communicating Sequential Processes*. Prentice Hall, 1985.

[14] K. G. Larsen and A. Skou. Bisimulation through probabilistic testing. *Information and Computation*, 94(1):1–28, 1991.

[15] R. Milner. *Communication and Concurrency*. Prentice Hall, 1989.

[16] F. Moller. *Axioms for Concurrency*. PhD thesis, Report CST-59-89, Department of Computer Science, University of Edinburgh, 1989.

[17] D. M. Park. Concurrency and automata on infinite sequences. In *Theoretical Computer Science, 5th GI-Conference*, volume 104 of *Lecture Notes in Computer Science*, pages 167–183. Springer, 1981.

[18] G. D. Plotkin. A structural approach to operational semantics. *Journal of Logic and Algebraic Programming*, 60–61:17–139, 2004.

[19] M. Voorhoeve and S. Mauw. Impossible futures and determinism. *Information Processing Letters*, 80(1):51–58, 2001.