

Some Remarks on Time Additivity in Timed Labelled Transition Systems

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In Definition 9.1 on page 164 of the textbook, we introduce the notion of timed labelled transition system and present three axioms, namely (9.2)–(9.4) on pages 164–165 of the book, that we expect those structures to satisfy. The reason for presenting those three axioms is that they impose natural requirements and define just about the properties that we need over timed labelled transition systems in the remainder of Part II of the book.

Note, however, that there are other possible choices of axioms for timed labelled transition systems one can consider. Moreover, there are some very natural axioms that we do not mention explicitly and that are satisfied, for instance, by the timed labelled transition systems giving semantics to timed automata. An example of such an axiom—which is imposed on timed labelled transition systems in, e.g., [1, Page 4]—is the following one:

$$\text{if } s \xrightarrow{d} s' \text{ and } s' \xrightarrow{d'} s'', \text{ for some states } s', s'', \text{ then } s \xrightarrow{d+d'} s''. \quad (1)$$

This requirement states the, intuitively very reasonable, fact that if a state s can delay d time units and then delay a further d' time units, then it can also delay $d + d'$ time units. Moreover, the effect of this single delay of duration $d + d'$ is the same as that of the two delays of duration d and d' .

We encourage you to think of the timing behaviour of the skeletal control program on page 160 of the textbook to see that requirement (1) makes a lot of sense. Moreover, you should have little trouble in solving the following exercise.

Exercise 1 *Let A be a timed automaton, as defined in Definition 10.4 on page 179 of the textbook. Prove that the timed transition system $T(A)$ generated by A affords property 1.* ♦

On the other hand, in [1] timed transition systems are not required to satisfy property (9.4) on page 165 of the textbook and may have nondeterministic delay transitions.

The treatment of delays of length zero in [1] is also slightly different from the one in our textbook. In that reference, the authors require a timed labelled transition system to satisfy our requirement (9.3) together with the following property:

$$\text{if } s \xrightarrow{0} s' \text{ and } s' \xrightarrow{0} s \text{ then } s = s'. \quad (2)$$

You should convince yourself that this property is satisfied by the timed labelled transition systems we consider in our textbook.

To sum up, there is nothing particularly canonical about the definition of timed labelled transition system presented in the textbook. Other definitions are possible and it is worth comparing the different formulations that have been presented in the literature. In particular, the mathematically-oriented readers will enjoy reading the comparison of additivity axioms for timed labelled transition systems presented in [1].

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References

- [1] A.S.A. Jeffrey, S.A. Schneider and F.W. Vaandrager. A comparison of additivity axioms in timed transition systems. Report CS-R9366, CWI, Amsterdam, November 1993. Computer Science Technical Report 93/11, School of Cognitive and Computing Sciences, University of Sussex, 1993. Available at <http://www.cwi.nl/ftp/CWIreports/AP/CS-R9366.ps.Z>.