Counting Temporal Logics

François Laroussinie (1)
Joint work with: Antoine Meyer (2), Eudes Petonnet (1)
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(1) LIAFA / Univ. Paris 7 – CNRS
(2) LIGM, Univ. Paris Est – CNRS

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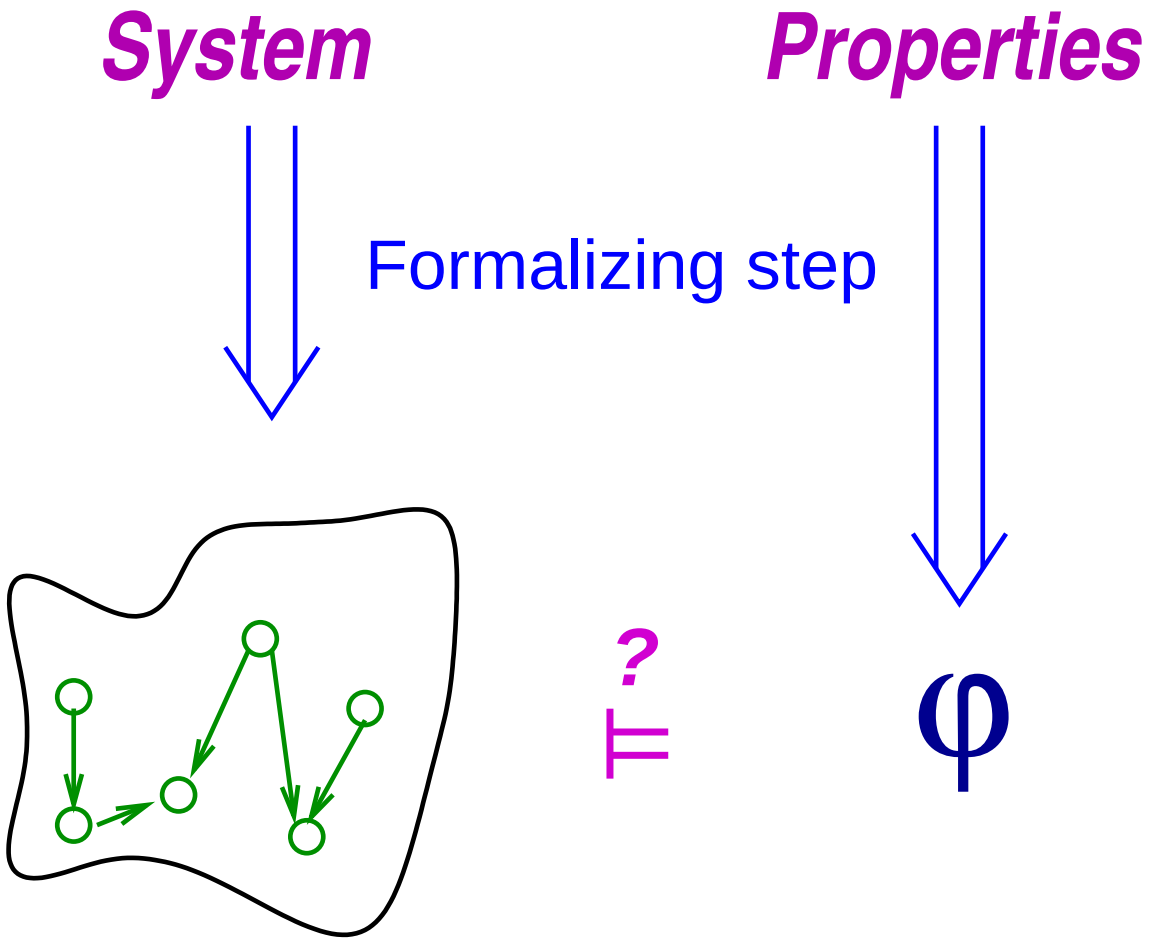
Model checking



Classical framework

The system is modeled with an automaton, a Kripke structure, a Petri net,...

The specification is stated with a Temporal Logic.

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Then just press the button ! (and wait...)

Choose the right models !

Temporal logics for the specification

Classical Temporal Logics (CTL, LTL,. . .) are very convenient to specify reactive systems

- good expressive power,
- natural semantics,
- succinctness,
- efficient (. . .) decision procedures (and tools !),
- nice extensions (timed TL, probabilistic TL,. . .).

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examples...

Linear-time Temporal Logic

Formulas built from AP, $\wedge$, $\neg$, U, X, S, X^{-1}, ...

- $\varphi \mathsf{U} \psi$: φ holds for every suffix until ψ is satisfied.
- $\mathsf{F}\varphi \stackrel{\text{def}}{=} \top \mathsf{U} \varphi$: "eventually φ"
- $\mathsf{G}\psi \stackrel{\text{def}}{=} \neg \mathsf{F} \neg \varphi$: "always φ"
- $\mathsf{X}\psi$: "The next state satisfies φ"

Formulas are interpreted over infinite executions: $q_0 \cdot q_1 \cdot q_2 \cdots$

$\mathcal{S} \models \Phi \Leftrightarrow \Phi$ holds for any run in $\mathcal{S}$.

- $\mathsf{G}(\texttt{Pb} \Rightarrow \mathsf{F}\,\texttt{Alarm})$
- $(\mathsf{GF}\,\texttt{P}_1) \Rightarrow (\mathsf{GF}\,\texttt{P}_2)$

Branching-time Temporal Logic

Formulas contain also path quantifiers: E and A.

Formulas are interpreted over states of Kripke structures.

$\mathcal{S} \models \Phi \Leftrightarrow \Phi$ holds for q_{init}.

- $\mathsf{E}\left(\mathtt{P}_1\ \mathsf{U}\ (\mathsf{AX}\ \mathtt{P}_2)\right)$
- $\mathsf{AG}(\mathtt{Pb} \Rightarrow \mathsf{AF}\ \mathtt{Alarm})$
- $\mathsf{AG}(\mathtt{Pb} \Rightarrow \mathsf{EF}\ \mathtt{Alarm})$

Counting. . .

Here our aim is to make the specifications of counting properties easy to write in TL.

Adding counting constraints in formulas

Consider an ATM.

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$$\neg \mathsf{E}\neg\texttt{error}\mathsf{U}\texttt{lock} \wedge$$
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$$\neg \mathsf{E}\neg\texttt{error}\mathsf{U}\Big(\texttt{error} \wedge \mathsf{EXE}\neg\texttt{error}\mathsf{U}(\texttt{error} \wedge \ldots \wedge \mathsf{EXE}\neg\texttt{error}\mathsf{U}\texttt{lock})\Big)$$

With counting constraints

"three mistakes forbid cash retrieval"

$$\neg \mathsf{EF}_{[\sharp \texttt{error} \geq 3]}\, \texttt{money}$$

"whenever the PIN is locked, at least three erroneous attempts have been made"

$$\neg \mathsf{EF}_{[\sharp \texttt{error} \leq 2]}\, \texttt{lock}$$

Counting in formulas

Other examples :

$\mathsf{EF}_{[\sharp \mathtt{EX Pb} < 2 \wedge \sharp \mathtt{ok} > 10]} \mathrm{P}$, $\mathsf{EF}_{[\sharp \mathtt{ok} - \sharp \mathtt{bad} > 10]} \mathrm{P}$, $\mathsf{AG}_{[10 \cdot \sharp \mathtt{ok} < 300 \cdot \sharp \mathtt{bad}]} \bot, \ldots$

Counting in formulas

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Objectives:
study the extensions of LTL and LTL with counting constraints of the form. . .

$$\bigwedge \bigvee \Big(\sum_{i=1}^{\ell} \alpha_i \cdot \sharp\varphi_i \;\sim k \Big)$$

with $\alpha_i \in \mathbb{Z}$. . . and all sensible restrictions. . .

Counting extensions of TL

- LTL with some regular expressions with quantitative constraints [ET97] $\rightsquigarrow$ exponential algo. in $|\Phi|$ and the *value* of constants.
- CTL with constraints (with parameters) [ET99].
  Constraints are positive BC of $\sum_i P_i \leq c$ with $P_i \in \mathrm{AP}$:
  - Model-checking E_U_ is shown to be NP-complete;
  - A polynomial algorithm is given for a restricted logic.
- A branching-time TL with general counting constraint (using freeze var.) is undecidable [YMW97].
- LTL and CTL with Presburger constraints [BEH95b, BEH95a] for some infinite state processes.
- Sugar/PSL [psl03]: extension of LTL.
- (timed extensions...)

Outline

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Kripke structures

CTL formulas are interpreted over the states of a Kripke structure:

AP = a set of atomic propositions.

$\mathcal{S} = \langle Q, q_0, \rightarrow, \ell \rangle$

- Q is finite set of states,
- $q_0 \in Q$ is the initial state,
- $\rightarrow \subseteq Q \times Q$ is total accessibility relation, and
- $\ell : Q \rightarrow 2^{\mathsf{AP}}$ is a labeling of states with atomic propositions.

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An execution: $\rho : s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \cdots$

$\rho(i) \stackrel{\text{def}}{=} s_i, \quad \rho^i \stackrel{\text{def}}{=} s_i \rightarrow s_{i+1} \cdots, \quad \rho_{|i} \stackrel{\text{def}}{=} s_0 \rightarrow \cdots \rightarrow s_i$

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no cost, no weight, no time . . . !

CTL with counting constraints

Definition

$$\text{CCTL} \ni \varphi, \psi \ ::= \ P \mid \varphi \wedge \psi \mid \neg\varphi \mid \mathsf{E}\varphi\mathsf{U}_{[C]}\psi \mid \mathsf{A}\varphi\mathsf{U}_{[C]}\psi$$

where $P \in \text{AP}$ and C is a constraint of the form:

$$\bigwedge \bigvee \left(\sum_{i=1}^{\ell} \alpha_i \cdot \sharp\varphi_i \sim k \right)$$

with $\varphi_i \in \text{CCTL}$

Semantics of CTL

$\mathcal{S} = \langle Q, q_0, \rightarrow, \ell \rangle$

Semantics

$$
\begin{array}{lll}
q \models P & \text{iff} & P \in \ell(q) \\
q \models \varphi \wedge \psi & \text{iff} & q \models \varphi \text{ and } q \models \psi \\
q \models \neg\varphi & \text{iff} & q \not\models \varphi \\
q \models \mathsf{EX}\varphi & \text{iff} & \exists q \rightarrow q', \text{s.t. } q' \models \varphi \\
q \models \mathsf{E}\varphi\mathsf{U}\psi & \text{iff} & \exists \rho \in \mathsf{Runs}(q), \text{s.t.} \exists k \geq 0, \rho(k) \models \psi, \text{ and} \\
 & & \forall 0 \leq i < k,\ \rho(i) \models \varphi \\
q \models \mathsf{A}\varphi\mathsf{U}\psi & \text{iff} & \forall \rho \in \mathsf{Runs}(q), \exists k \geq 0, \text{s.t. } \rho(k) \models \psi, \text{ and} \\
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Semantics

$$q \models \mathsf{E}\varphi\mathsf{U}_{[C]}\psi \quad \text{iff} \quad \exists \rho \in \mathsf{Runs}(q),\ \exists k \geq 0, \rho(k) \models \psi,\ \rho_{|k-1} \models C,\ \text{and } \forall 0 \leq i < k,\ \rho(i) \models \varphi$$

$$q \models \mathsf{A}\varphi\mathsf{U}_{[C]}\psi \quad \text{iff} \quad \forall \rho \in \mathsf{Runs}(q),\ \exists k \geq 0,\ \rho(k) \models \psi,\ \rho_{|k-1} \models C,\ \text{and } \forall 0 \leq i < k,\ \rho(i) \models \varphi$$

Semantics of constraints Let π be a finite run.

$\pi \models C$ is based on the interpretation of $\sharp\varphi$ over π: $\pi[\sharp\varphi]$

$$\pi[\sharp\varphi] \stackrel{\text{def}}{=} |\{j \mid 0 \leq j \leq |\pi| \ \wedge\ \pi(j) \models \varphi\}|$$

Abbreviations. . .

- "It is possible to reach Φ and C":

$$\mathsf{EF}_{[C]}\,\Phi \stackrel{\text{def}}{=} \mathsf{E}\top\,\mathsf{U}_{[C]}\,\Phi$$

- "Along every path, one can reach Φ and C":

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- "Always: C implies Φ":

$$\mathsf{AG}_{[C]}\,\Phi \stackrel{\text{def}}{=} \neg\mathsf{EF}_{[C]}\neg\Phi$$

- "There is a path along which C implies Φ":

$$\mathsf{EG}_{[C]}\,\Phi \stackrel{\text{def}}{=} \neg\mathsf{AF}_{[C]}\neg\Phi$$

Examples of formulas

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- we can reach the alarm state in less than 50 transitions:

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- $\mathsf{E}\varphi\mathsf{U}_{[c]}\psi \stackrel{\text{def}}{=} \mathsf{EF}_{[c\wedge\sharp(\neg\varphi)=0]}\psi$
- we can reach the alarm state in less than 50 transitions:

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- we can reach the alarm state in less than 50 "ticks":

$$\mathsf{EF}_{[\sharp\mathtt{tick}<50]}\mathtt{Alarm}$$

Timed CTL over KSs with tick: $\mathsf{E}\varphi\mathsf{U}_{<k}\psi \stackrel{\text{def}}{=} \mathsf{E}\varphi\mathsf{U}_{[\sharp\mathtt{tick}<k]}\psi$

Examples of formulas

- For an ATM: “it is not possible to get money when three mistakes are made in the same session”:

$$\mathsf{AG}\big(\neg \mathsf{EF}_{[\sharp \texttt{error} \geq 3 \wedge \sharp \texttt{reset} = 0]} \texttt{money}\big)$$

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- $\mathsf{AG}\,(\,\mathsf{EF}_{[\sharp(\mathsf{EX}\mathtt{alarm}) \leq 5]}\mathtt{init}\,)$
  "It is always possible to reach `init` along a path where less than 5 states have an `alarm` state as successor."

Examples of formulas

- The bounded waiting property with bound 10 for a mutual exclusion algorithm with n processes:

$$\mathsf{AG} \bigwedge_i \left(\mathtt{request}_i \Rightarrow \neg \mathsf{EF}_{[\sum_{j \neq i} \sharp \mathtt{CS}_j > 10 \wedge \sharp \mathtt{CS}_i = 0]} \top \right)$$

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- "The number of receive events can not exceed the number of send events":

$$\mathsf{AG}_{[\sharp \mathtt{send} - \sharp \mathtt{receive} < 0]} \bot$$

Examples of formulas

- Quantitative fairness: "the φ_i's occur infinitely often along every run and there is no sub-run where φ_i holds for more than 10 states and φ_j holds for less than 4 states":

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- Ratio: "there is a path leading to P s.t. the ratio of `error` states is less than 1 percent."

$$\mathsf{EF}_{[100\cdot\sharp\texttt{error}-\sharp\top<0]}P$$

(Constraints "$\frac{\sharp P}{\sharp P'} \sim k$" can easily be expressed.)

Outline

Expressiveness

Constraints: $\bigwedge \bigvee \Sigma_{i=1}^{l} \alpha_i \cdot \sharp\varphi_i \sim k$

Proposition

CCTL with positive coefficients can be translated into CTL.

Idea: count manually occurrences of events using nested U modalities. . . and consider all possible shuffles of such occurrences.

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Proposition

The formula $\varphi = \mathsf{AG}_{[\sharp A - \sharp B < 0]} \bot$ cannot be translated into CTL.

Idea: the set of models of any CTL formula can be recognized by a Büchi alternating tree automaton. For φ, we need a counter for any path.

Diagonal constraints increase the expressive power of CTL.

Succinctness

CCTL formulas with constraints $\bigwedge \bigvee \Sigma_{i=1}^{l} \alpha_i \cdot \sharp \varphi_i \sim k$ with positive coefficients can be translated into CTL, but in these constraints, there are three potential sources of concision:

- the binary encoding of constants,
- the Boolean combinations in the constraints,
- and the sums.

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- the binary encoding of constants,
- the Boolean combinations in the constraints,
- and the sums.

Only the two first yield an exponential improvement in succinctness.

Succinctness – binary encoding of the constants

The natural translation of $\mathrm{EF}_{[\sharp A=k]}B$ into CTL yields an exponential formula.
(it uses k nested modalities. . .)

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Proposition CCTL with constraints $\sharp\varphi \sim k$ can be exponentially more succinct than CTL.

Idea: TCTL formulas $\mathrm{EF}_{<k}A$ and $\mathrm{EF}_{>k}A$ do not admit any equivalent CTL formula of temporal height less than k [LST03].

Succinctness – Boolean combinations

Proposition CCTL with constraints $\bigwedge(\sharp\varphi_i \sim k_i)$ and unary encoding of integers can be exponentially more succinct than CTL.

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Idea: any CTL formula equivalent to ψ:

$$\psi \;=\; \mathsf{E}(\mathsf{F}\, P_0 \wedge \ldots \wedge \mathsf{F}\, P_n)$$

must be of length exponential in n [Wil99, Al03].

$$\psi \;\equiv\; \mathsf{EF}_{[\bigwedge_i \sharp P_i \geq 1]}\top$$

(no need of binary encoding of constants)

Succinctness – Sums

Proposition For every formula $\Phi \in$ CCTL with constraints of the form $\sum \sharp\varphi_i \sim k$ and with unary encoding of integers, there exists an equivalent CTL formula of DAG-size polynomial in $|\Phi|$.

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Example: $\Phi = \mathsf{EF}_{\sum_i \sharp P_i = K} A$ is equivalent to Ψ_K with:

$$\begin{aligned}
\Psi_k &\stackrel{\text{def}}{=} \mathsf{E}\Big(\bigwedge_i \bar{P}_i\Big)\mathsf{U}\Big(\bigvee_i P_i \wedge \beta_{k,1,\bot}\Big) && (k > 0)\\
\Psi_0 &\stackrel{\text{def}}{=} \mathsf{E}\Big(\bigwedge_i \bar{P}_i\Big)\mathsf{U}\, A\\
\Psi_{-1} &\stackrel{\text{def}}{=} \bot
\end{aligned}$$

$$\begin{aligned}
\beta_{k,i,\epsilon} &\stackrel{\text{def}}{=} (P_i \wedge \beta_{k-1,i+1,\top}) \vee (\bar{P}_i \wedge \beta_{k,i+1,\epsilon}) && (i < n)\\
\beta_{k,n,\top} &\stackrel{\text{def}}{=} (P_n \wedge \mathsf{EX}\,\Psi_{k-1}) \vee (\bar{P}_n \wedge \mathsf{EX}\,\Psi_k)\\
\beta_{k,n,\top} &\stackrel{\text{def}}{=} P_n \wedge \mathsf{EX}\,\Psi_{k-1}
\end{aligned}$$

Comparison with Past

Counting constraints deal with past events !

We could use past-time modalities:

$$\mathsf{AG}\Big(\texttt{money} \Rightarrow \neg\, \mathsf{F}_{s}^{-1}(\texttt{error} \wedge \mathsf{F}_{s}^{-1}(\texttt{error} \wedge \mathsf{F}_{s}^{-1}\texttt{error}))\Big)$$

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+ Past-time modalities allow us to express properties over the ordering of the events.
+ They (often) increase the expressive power (compared to CTL).
+ Boolean combinations are directly handled...

– Counting constraints are still more succinct.
– Complexity (model-checking CTL + F^{-1} is PSPACE-Complete)

Outline

Model checking algorithms

The complexity of the model checking problem depends on the type of the constraints.
Complexity ranges from P-complete to. . . undecidability.

Constraints

Given $l, k \in \mathbb{N}$, $k' \in \mathbb{Z}$ and $\sim \in \{<, \leq, =, \geq, >\}$, we define:

$$\mathcal{C}_0: \quad \sharp\varphi \sim k$$

$$\mathcal{C}_1: \quad \Big(\sum_{i=1}^{l} \sharp\varphi_i\Big) \sim k$$

$$\alpha\mathcal{C}_1: \quad \Big(\sum_{i=1}^{l} \alpha_i \cdot \sharp\varphi_i\Big) \sim k \qquad \alpha_i \in \mathbb{N}$$

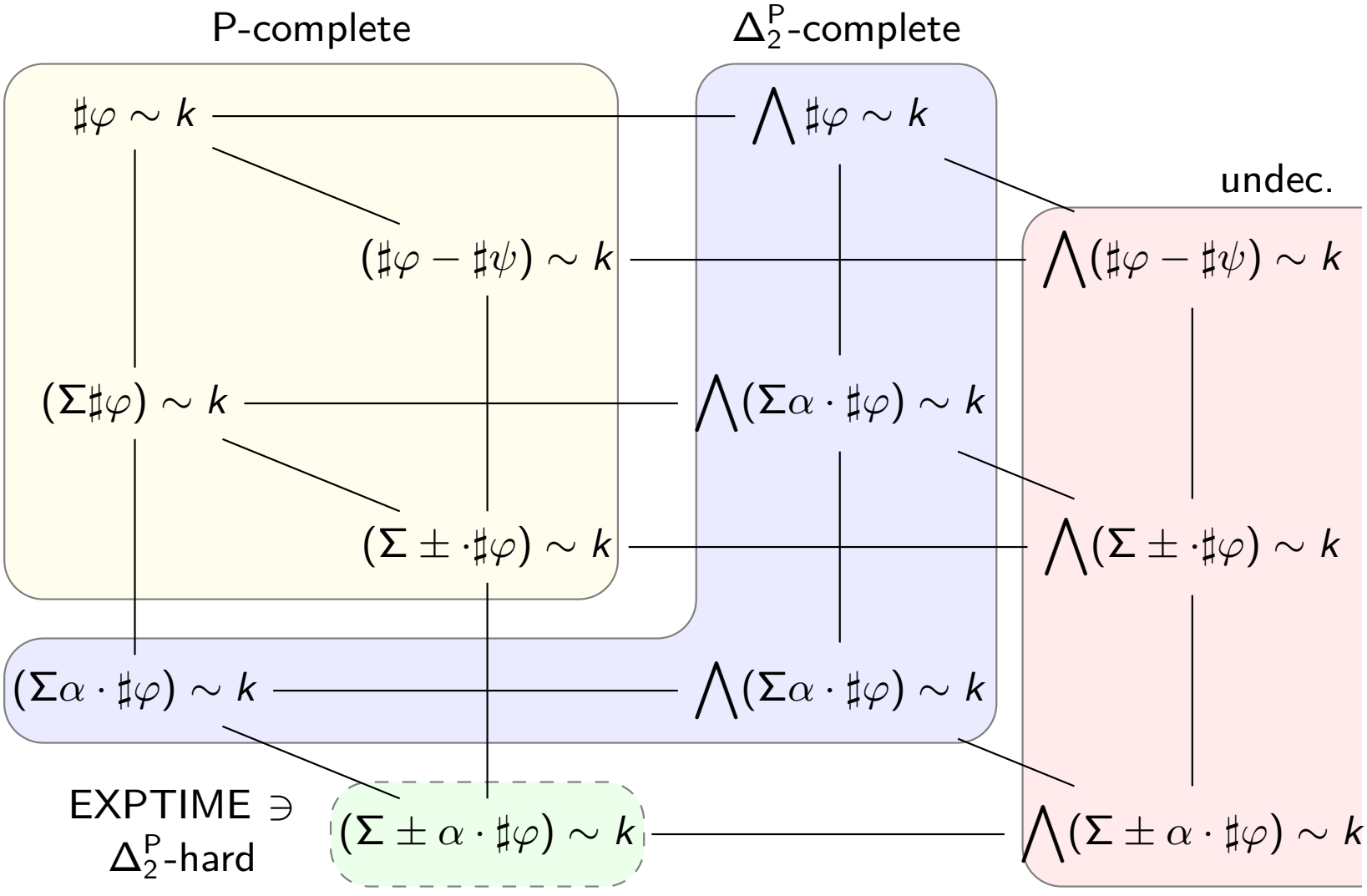
$$\mathcal{C}_2: \quad (\sharp\varphi - \sharp\psi) \sim k'$$

$$\mathcal{C}_3: \quad \Big(\sum_{i=1}^{l} \pm \cdot \sharp\varphi_i\Big) \sim k'$$

$$\alpha\mathcal{C}_3: \quad \Big(\sum_{i=1}^{l} \beta_i \cdot \sharp\varphi_i\Big) \sim k' \qquad \beta_i \in \mathbb{Z}$$

And $\mathcal{B}(\mathcal{C})$ is the set of Boolean combinations of constr. in $\mathcal{C}$.

Conclusion


P-complete
Δ_2^{P}-complete
undec.
$\sharp\varphi \sim k$
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$(\sharp\varphi - \sharp\psi) \sim k$
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EXPTIME $\ni$
Δ_2^{P}-hard
$(\Sigma \pm \alpha \cdot \sharp\varphi) \sim k$
$\bigwedge(\Sigma \pm \alpha \cdot \sharp\varphi) \sim k$

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Model checking $\mathrm{CCTL}_{\mathcal{C}_1}$: $(\Sigma \sharp \varphi_i) \sim k$

Theorem Model-checking $\mathrm{CCTL}_{\mathcal{C}_1}$ is P-complete.

Reduction to a model-checking problem for TCTL formula over Kripke structures with $0/1$ durations.

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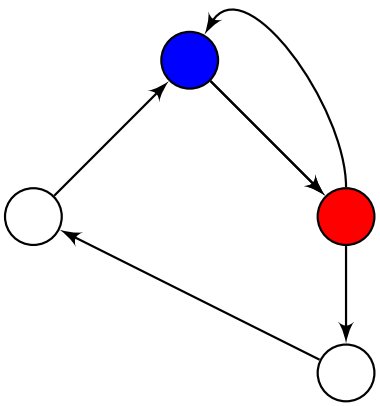
Reduction to a model-checking problem for TCTL formula over Kripke structures with 0/1 durations.

Example: $\mathcal{S} = (Q, R, \ell)$, and $\Phi = \mathsf{E}\varphi\mathsf{U}_{[\sharp P_1 + \sharp P_2 \sim k]}\psi$

$\models P_1$ $\models P_1 \wedge P_2$



Model checking $\mathrm{CCTL}_{\mathcal{C}_1}$: $(\Sigma \sharp \varphi_i) \sim k$

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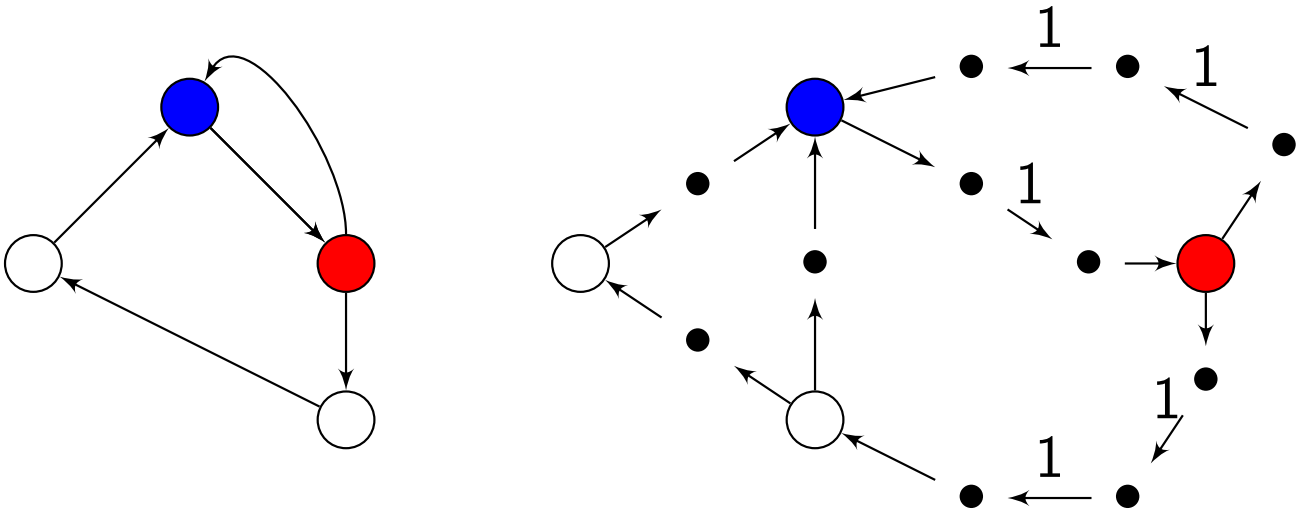
Reduction to a model-checking problem for TCTL formula over Kripke structures with 0/1 durations.

Example: $\mathcal{S} = (Q, R, \ell)$, and $\Phi = \mathsf{E}\varphi \mathsf{U}_{[\sharp P_1 + \sharp P_2 \sim k]}\psi$

$\bullet \models P_1$ $\qquad$ $\bullet \models P_1 \wedge P_2$



$\Phi' = \mathsf{E}(\neg \bullet \Rightarrow \varphi)\mathsf{U}_{\sim k}(\neg \bullet \wedge \psi) \in \mathrm{TCTL}$

Model-checking $\mathrm{CCTL}_{\mathcal{C}_3}$

For $\mathrm{CCTL}_{\mathcal{C}_3}$ (*i.e.* with constraint of the form "$(\Sigma_{i=1}^{l} \pm \cdot \sharp\varphi_i) \sim k$'"), the previous encoding gives a DKS with durations in $\{-1, 0, 1\}$:

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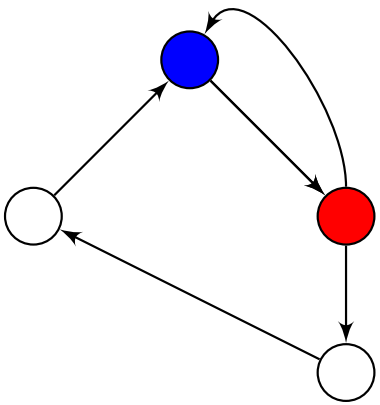
$\models P_1$ $\models P_2$



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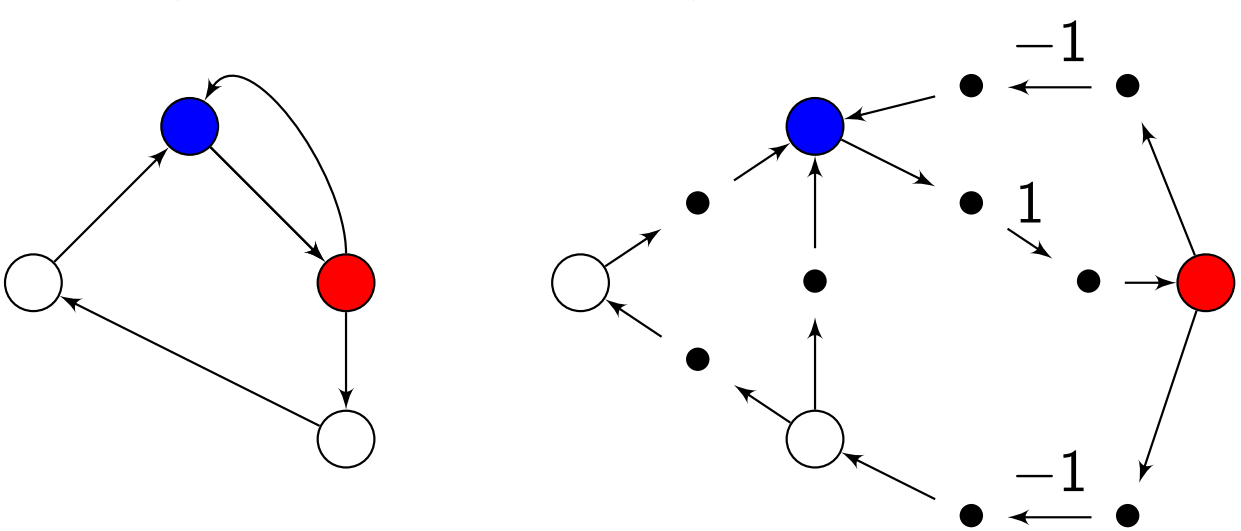

Reduction to a model-checking problem for TCTL over $\mathrm{DKS}^{-1/0/1}$.

Model-checking TCTL over $\mathrm{DKS}^{-1/0/1}$

Theorem

Model-checking TCTL over $\mathrm{DKS}^{-1/0/1}$ is P-complete.

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- $\Phi \stackrel{\text{def}}{=} \mathrm{E}\varphi\mathrm{U}_{=k}\psi$:

Compute $R_k \stackrel{\text{def}}{=} \{(q, q') \in S^2 \mid \exists q \xrightarrow{k} q'\}$ as follows:

- $R_k = R_{\lfloor k/2 \rfloor} \cdot R_{\lfloor k/2 \rfloor} \cdot R_{k\%2}$
- $R_1 \stackrel{\text{def}}{=} R_0 \cdot \xrightarrow{1} \cdot R_0$
- R_0 is the least solution of:

$$X = (\xrightarrow{0})^* \cup X \cdot (\xrightarrow{1} \cdot X \cdot \xrightarrow{-1} \cup \xrightarrow{-1} \cdot X \cdot \xrightarrow{1}) \cdot X$$

Add constraints for $\varphi \ldots \rightsquigarrow R'_k$

$\Rightarrow q \models \Phi$ iff $(q, q') \in R'_k$ for some q' satisfying ψ'.

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Model-checking $\mathrm{CCTL}_{\mathcal{C}_3}$: $(\Sigma \pm \cdot \sharp \varphi_i) \sim k$

Corollary Model-checking $\mathrm{CCTL}_{\mathcal{C}_3}$ is P-complete.

Outline

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_0)}$: $\bigwedge \bigvee \sharp\varphi \sim k$

Theorem

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_0)}$ is Δ_2^{P}-hard.

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Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_0)}$ is Δ_2^{P}-hard.

Reduction from SNSAT:

$$\begin{cases} z_1 \stackrel{\text{def}}{=} \exists X_1.\varphi_1(X_1) \\ z_2 \stackrel{\text{def}}{=} \exists X_2.\varphi_2(z_1, X_2) \\ \qquad\qquad \dots \\ z_n \stackrel{\text{def}}{=} \exists X_n.\varphi_n(z_1, \dots, z_{n-1}, X_n) \end{cases}$$

Question: $z_n = \top$?

Model-checking $\mathrm{CCTL}_{\alpha\mathcal{C}_1}$: $(\Sigma_{i=1}^{l}\alpha_i \cdot \sharp\varphi_i) \sim k,\ \alpha_i \in \mathbb{N}$

Theorem

Model-checking $\mathrm{CCTL}_{\alpha\mathcal{C}_1}$ is Δ_2^{P}-hard.

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Theorem

Model-checking $\mathrm{CCTL}_{\alpha\mathcal{C}_1}$ is Δ_2^{P}-hard.

Reduction from the MC pb for TCTL over KSs with durations.

Let $\mathcal{S} = (Q, R_{\mathcal{S}}, \ell)$ be a DKS.

"$q \xrightarrow{k} q'$ in $\mathcal{S}$" is replaced by "$q(\mathtt{ok}) \to q_{aux}(P_k) \to q'(\mathtt{ok})$"

The TCTL formula $\Phi \stackrel{\text{def}}{=} \mathrm{E}\varphi\mathrm{U}_{\sim m}\psi$ is replaced by $\widetilde{\Phi}$:

$$\mathrm{E}(\mathtt{ok} \Rightarrow \widetilde{\varphi})\ \mathrm{U}_{[C]}\ (\mathtt{ok} \wedge \widetilde{\psi})$$

with $C \stackrel{\text{def}}{=} \sum_{d \in W} d \cdot \sharp P_d \sim m$.

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$: $\bigvee\bigwedge(\Sigma_{i=1}^{l}\alpha_i \cdot \sharp\varphi_i) \sim k$, $\alpha_i \in \mathbb{N}$

Theorem

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$ is in Δ_2^{P}.

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Theorem

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$ is in Δ_2^{P}.

Based on the Parikh image of the runs satisfying $\mathrm{EF}_{[C]}\psi$.

- we can assume that $|\rho|$ is in $O(|Q| \cdot 2^{|C|})$;
- check in polynomial time that a guessed Parikh image corresponds to some path;
- verify that it verifies the formula.

And for $\mathrm{EG}_{[C]}\psi$, we look for an infinite run but $(\Sigma\alpha_i \cdot \sharp\varphi_i) \sim k$ may change its truth value at most twice along the run...

Outline

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$: $\bigwedge \bigvee (\sharp\varphi - \sharp\psi) \sim k$

Theorem

The model-checking problem for $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$ is undecidable.

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$: $\bigwedge\bigvee(\sharp\varphi - \sharp\psi) \sim k$

Theorem

The model-checking problem for $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$ is undecidable.

Reduction from the halting problem of a two-counter machine $\mathcal{M}$.

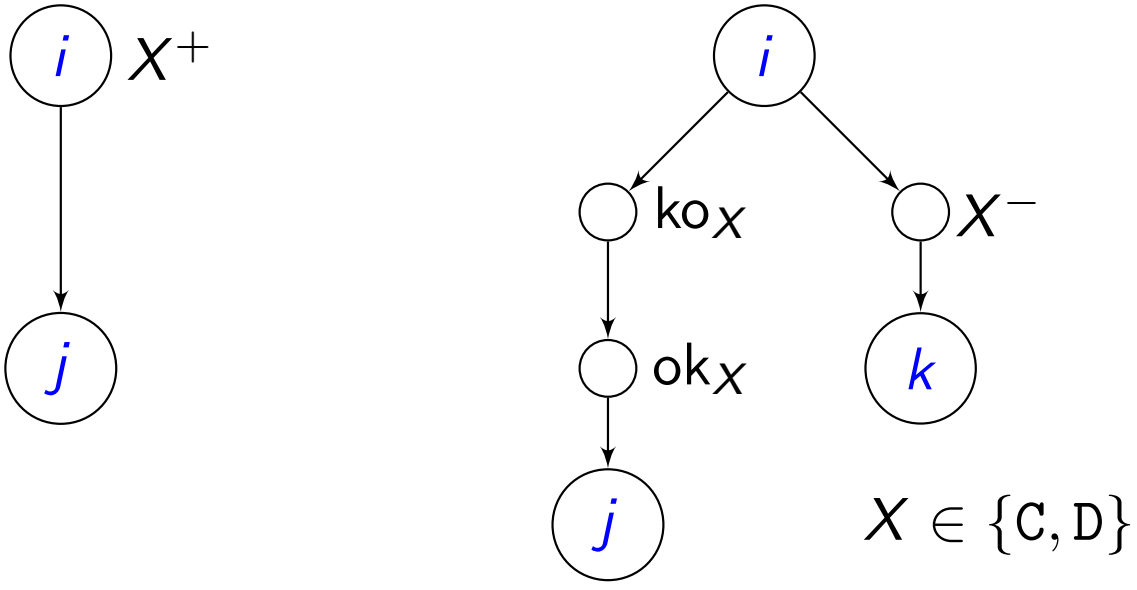
Two counters: `C` and `D`, and 5 kinds of instructions:

- i : `C++, goto` j
- i : `D++, goto` j
- i : `if C=0 then goto` j `else C--, goto` k
- i : `if D=0 then goto` j `else D--, goto` k
- i : `halt`

Model-checking $\mathrm{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$: $\bigwedge\bigvee(\sharp\varphi - \sharp\psi) \sim k$

From $\mathcal{M}$, we build the following Kripke structure:

i : `X++, goto` j $\quad$ i : `if X=0 then goto` j `else X--, goto` k



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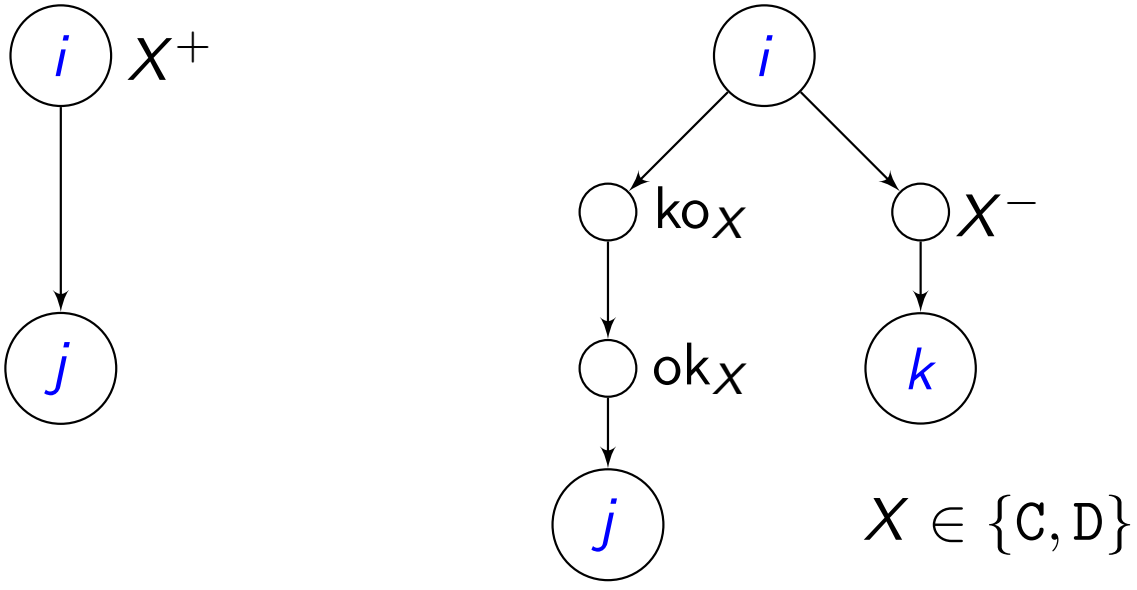
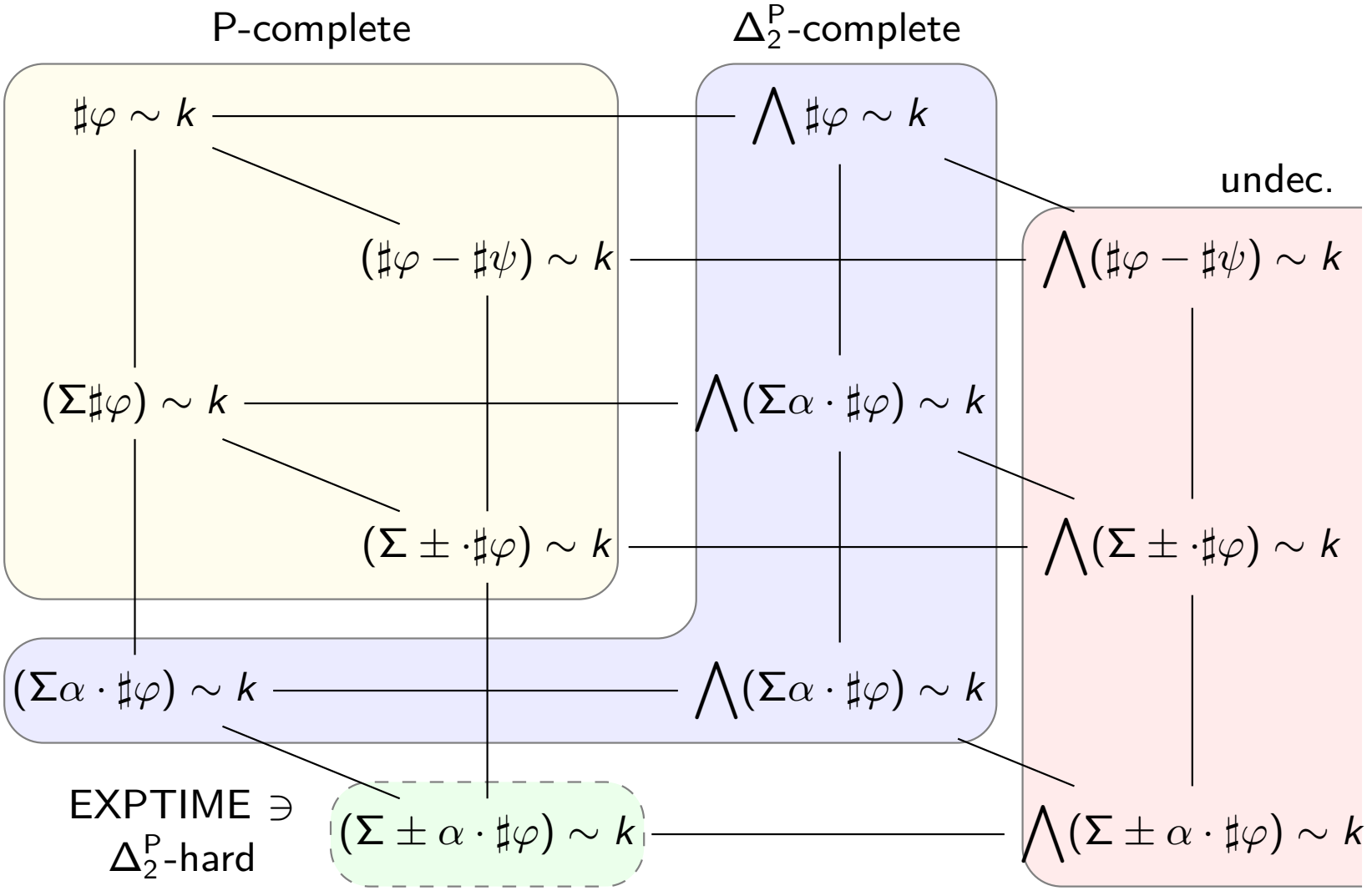


$\mathcal{M}$ does not halt *if and only if* $q_1 \models_{\mathcal{S}_\mathcal{M}} \mathsf{EG}_{[C]}\bot$ with:

$$C \stackrel{\text{def}}{=} (\sharp\mathsf{halt} \geq 1) \ \vee C_{bad}$$

$$C_{bad} \stackrel{\text{def}}{=} \bigvee_{X\in\{\mathtt{C},\mathtt{D}\}} \Big((\sharp X^+ - \sharp X^- < 0) \vee (\sharp X^+ - \sharp X^- > 0 \wedge \sharp \mathsf{ko}_X - \sharp \mathsf{ok}_X > 0)\Big)$$

Conclusion


P-complete
Δ_2^{P}-complete
undec.
$\sharp\varphi \sim k$
$\bigwedge \sharp\varphi \sim k$
$(\sharp\varphi - \sharp\psi) \sim k$
$\bigwedge(\sharp\varphi - \sharp\psi) \sim k$
$(\Sigma\sharp\varphi) \sim k$
$\bigwedge(\Sigma\alpha \cdot \sharp\varphi) \sim k$
$(\Sigma \pm \cdot\sharp\varphi) \sim k$
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EXPTIME $\ni$
Δ_2^{P}-hard
$(\Sigma \pm \alpha \cdot \sharp\varphi) \sim k$
$\bigwedge(\Sigma \pm \alpha \cdot \sharp\varphi) \sim k$

Outline

LTL + counting constraints

Syntax

$$LTL \ni \varphi, \psi \ ::= \ P \mid \varphi \wedge \psi \mid \neg\varphi \mid \varphi \mathsf{U} \psi \mid \mathsf{X}\varphi$$

where $P \in \mathsf{AP}$.

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Semantics $\rho : q_1 \to q_2 \to \cdots$

$$\begin{array}{lll}
\rho \models P & \text{iff} & P \in \ell(q_1) \\
\rho \models \varphi \wedge \psi & \text{iff} & \rho \models \varphi \text{ and } \rho \models \psi \\
\rho \models \neg\varphi & \text{iff} & \rho \not\models \varphi \\
\rho \models \mathsf{X}\varphi & \text{iff} & \rho^1 \models \varphi \\
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\rho \models \varphi \mathsf{U}_{[C]} \psi & \text{iff} & \exists k \geq 0, \rho^k \models \psi, \ \ \rho, k-1 \models C \\
 & & \text{and } \forall 0 \leq i < k, \ \rho^i \models \varphi
\end{array}$$

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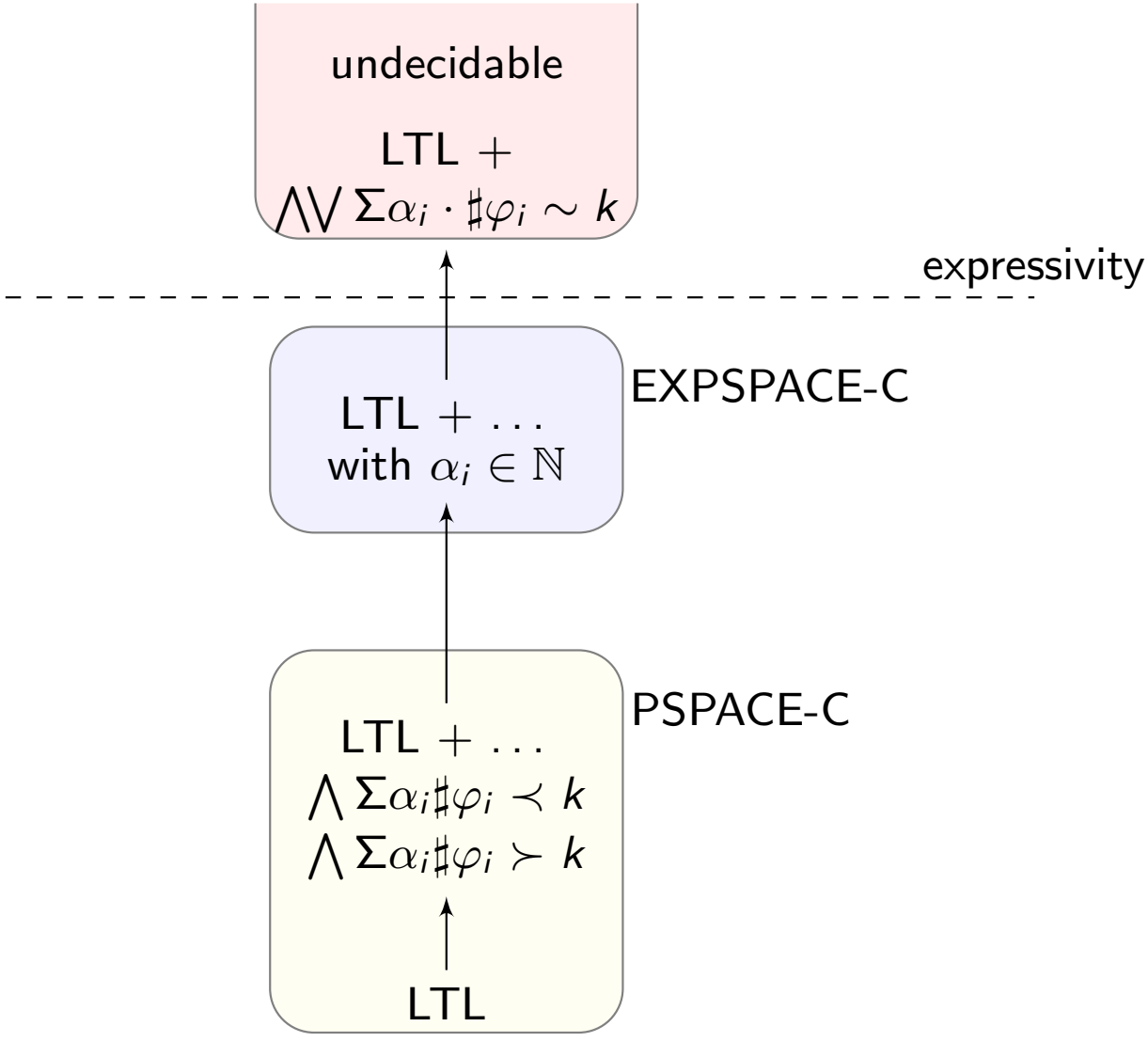
Overview of the results for LTL



Linear-time case

CLTL $\stackrel{\text{def}}{=}$ *LTL*+ constraints with "positive" coefficients.
Use standard techniques:

- alternating Büchi automata to recognize the models of a formula.
  (for CLTL, $\mathcal{A}_\Phi$ is exponential in $|\Phi|$.)
- satisfiability and model-checking
- removing equality makes procedures more efficient.
- couting along a unique path is easier...

Outline

Conclusion

- Other semantics are possible. . . For example, the cumulative semantics.
  $\mathrm{EF}^{c}_{[\sharp\top \leq k_1]}(P_1 \wedge \mathrm{EF}^{c}_{[\sharp\top \leq k_2]} P_2)$: "there is a run with less than k_2 transitions leading to P_2 and along this run there is some P_1 located at less than k_1 transitions from the initial state"

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- See in practice.
- Consider other operators (modulo,...).
- Consider properties over infinite runs (ratio).


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