

Counting Temporal Logics

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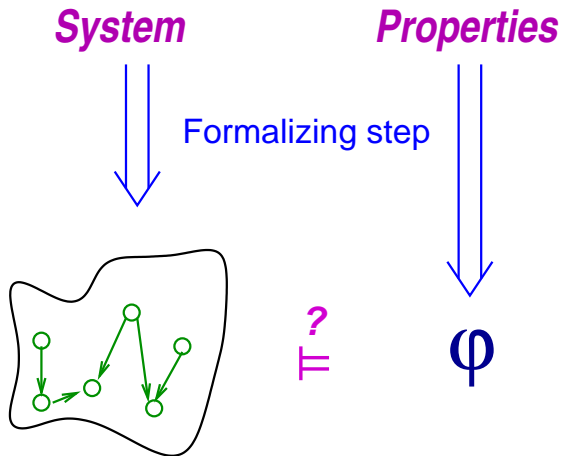
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Model checking



Classical framework

The **system** is modeled with an automaton, a Kripke structure, a Petri net, . . .

The **specification** is stated with a Temporal Logic.

The correctness of the system corresponds to a set of TL formulas that have to be satisfied by the model.

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Then just press the button ! (and wait. . .)

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Then just press the button ! (and wait. . .)

Choose the right models !

Temporal logics for the specification

Classical Temporal Logics (CTL, LTL,...) are very convenient to specify **reactive systems**

- good expressive power,
- natural semantics,
- succinctness,
- efficient (...) decision procedures (and tools !),
- nice extensions (timed TL, probabilistic TL,...).

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examples...

Linear-time Temporal Logic

Formulas built from AP, \wedge , \neg , U, X, S, X^{-1} , ...

- $\varphi U \psi$: φ holds for every suffix until ψ is satisfied.
- $F\varphi \stackrel{\text{def}}{=} \top U \varphi$: “eventually φ ”
- $G\varphi \stackrel{\text{def}}{=} \neg F \neg \varphi$: “always φ ”
- $X\psi$: “The next state satisfies φ ”

Formulas are interpreted over infinite **executions**: $q_0 \cdot q_1 \cdot q_2 \cdots$

$\mathcal{S} \models \Phi \Leftrightarrow \Phi$ holds for any run in \mathcal{S} .

- $G(\text{Pb} \Rightarrow F \text{Alarm})$
- $(GF P_1) \Rightarrow (GF P_2)$

Branching-time Temporal Logic

Formulas contain also **path quantifiers**: E and A.

Formulas are interpreted over **states** of Kripke structures.

$\mathcal{S} \models \Phi \Leftrightarrow \Phi$ holds for q_{init} .

- $E (P_1 U (AX P_2))$
- $AG(Pb \Rightarrow AF Alarm)$
- $AG(Pb \Rightarrow EF Alarm)$

Here our aim is to make the specifications of counting properties easy to write in TL.

Adding counting constraints in formulas

Consider an ATM.

“three mistakes forbid cash retrieval”

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“whenever the PIN is locked, at least three erroneous attempts have been made”

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“whenever the PIN is locked, at least three erroneous attempts have been made”

$$\begin{aligned} & \neg E \neg \text{error} U \text{lock} \wedge \\ & \neg E \neg \text{error} U \left(\text{error} \wedge EXE \neg \text{error} U \text{lock} \right) \wedge \\ & \neg E \neg \text{error} U \left(\text{error} \wedge EXE \neg \text{error} U \left(\text{error} \wedge \dots \right. \right. \\ & \qquad \qquad \qquad \left. \left. \wedge EXE \neg \text{error} U \text{lock} \right) \right) \end{aligned}$$

With counting constraints

“three mistakes forbid cash retrieval”

$$\neg \text{EF}_{[\#\text{error} \geq 3]} \text{money}$$

“whenever the PIN is locked, at least three erroneous attempts have been made”

$$\neg \text{EF}_{[\#\text{error} \leq 2]} \text{lock}$$

Counting in formulas

Other examples :

$EF_{[\#EX\#b < 2 \wedge \#ok > 10]}P$, $EF_{[\#ok - \#bad > 10]}P$, $AG_{[10 \cdot \#ok < 300 \cdot \#bad]} \perp, \dots$

Counting in formulas

Other examples :

$EF_{[\#EX\text{pb}<2 \wedge \#ok>10]}P$, $EF_{[\#ok-\#bad>10]}P$, $AG_{[10\cdot\#ok<300\cdot\#bad]} \perp, \dots$

Objectives:

study the extensions of LTL and LTL with counting constraints of the form...

$$\bigwedge \bigvee \left(\sum_{i=1}^{\ell} \alpha_i \cdot \#\varphi_i \sim k \right)$$

with $\alpha_i \in \mathbb{Z} \dots$ and all sensible restrictions...

Counting extensions of TL

- LTL with some regular expressions with quantitative constraints [ET97] \rightsquigarrow exponential algo. in $|\Phi|$ and the *value of constants*.
- CTL with constraints (with parameters) [ET99].
Constraints are positive BC of $\sum_i P_i \leq c$ with $P_i \in AP$:
 - Model-checking E_U_ is shown to be NP-complete;
 - A polynomial algorithm is given for a restricted logic.
- A branching-time TL with general counting constraint (using freeze var.) is **undecidable** [YMW97].
- LTL and CTL with Presburger constraints [BEH95b, BEH95a] for some infinite state processes.
- Sugar/PSL [psl03]: extension of LTL.
- (timed extensions. . .)

Outline

- 1 CCTL
- 2 Expressivity
- 3 Model checking
 - Efficient model-checking
 - and harder. . .
 - and even worst !
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Kripke structures

CTL formulas are interpreted over the states of a Kripke structure:

AP = a set of atomic propositions.

$$\mathcal{S} = \langle Q, q_0, \rightarrow, \ell \rangle$$

- Q is finite set of states,
- $q_0 \in Q$ is the initial state,
- $\rightarrow \subseteq Q \times Q$ is total accessibility relation, and
- $\ell : Q \rightarrow 2^{\text{AP}}$ is a labeling of states with atomic propositions.

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An execution: $\rho : s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \dots$

$$\rho(i) \stackrel{\text{def}}{=} s_i, \quad \rho^j \stackrel{\text{def}}{=} s_i \rightarrow s_{i+1} \dots, \quad \rho|_i \stackrel{\text{def}}{=} s_0 \rightarrow \dots \rightarrow s_i$$

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no cost, no weight, no time ...!

CTL with counting constraints

Definition

$$\text{CCTL} \ni \varphi, \psi ::= P \mid \varphi \wedge \psi \mid \neg \varphi \mid E\varphi U_{[C]}\psi \mid A\varphi U_{[C]}\psi$$

where $P \in \text{AP}$ and C is a constraint of the form:

$$\bigwedge \bigvee \left(\sum_{i=1}^{\ell} \alpha_i \cdot \#\varphi_i \sim k \right)$$

with $\varphi_i \in \text{CCTL}$

Semantics of CTL

$$\mathcal{S} = \langle Q, q_0, \rightarrow, \ell \rangle$$

Semantics

$q \models P$	iff	$P \in \ell(q)$
$q \models \varphi \wedge \psi$	iff	$q \models \varphi$ and $q \models \psi$
$q \models \neg\varphi$	iff	$q \not\models \varphi$
$q \models \text{EX}\varphi$	iff	$\exists q' \rightarrow q', \text{s.t. } q' \models \varphi$
$q \models \text{E}\varphi\text{U}\psi$	iff	$\exists \rho \in \text{Runs}(q), \text{s.t. } \exists k \geq 0, \rho(k) \models \psi, \text{ and}$ $\forall 0 \leq i < k, \rho(i) \models \varphi$
$q \models \text{A}\varphi\text{U}\psi$	iff	$\forall \rho \in \text{Runs}(q), \exists k \geq 0, \text{s.t. } \rho(k) \models \psi, \text{ and}$ $\forall 0 \leq i < k, \rho(i) \models \varphi$

CTL with counting constraints

Semantics

$q \models E\varphi U_{[C]}\psi$ iff $\exists \rho \in \text{Runs}(q), \exists k \geq 0, \rho(k) \models \psi, \rho|_{k-1} \models C,$
and $\forall 0 \leq i < k, \rho(i) \models \varphi$

$q \models A\varphi U_{[C]}\psi$ iff $\forall \rho \in \text{Runs}(q), \exists k \geq 0, \rho(k) \models \psi, \rho|_{k-1} \models C,$
and $\forall 0 \leq i < k, \rho(i) \models \varphi$

Semantics of constraints Let π be a finite run.

$\pi \models C$ is based on the interpretation of $\#\varphi$ over π : $\pi[\#\varphi]$

$$\pi[\#\varphi] \stackrel{\text{def}}{=} \{j \mid 0 \leq j \leq |\pi| \wedge \pi(j) \models \varphi\}$$

Abbreviations. . .

- “It is possible to reach Φ and C ”:

$$EF_{[C]} \Phi \stackrel{\text{def}}{=} E \top U_{[C]} \Phi$$

- “Along every path, one can reach Φ and C ”:

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Abbreviations. . .

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- “Always: C implies Φ ”:

$$AG_{[C]} \Phi \stackrel{\text{def}}{=} \neg EF_{[C]} \neg \Phi$$

- “There is a path along which C implies Φ ”:

$$EG_{[C]} \Phi \stackrel{\text{def}}{=} \neg AF_{[C]} \neg \Phi$$

Examples of formulas

- $EX \varphi \stackrel{\text{def}}{=} EF_{[\#T=1]} \varphi$

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- we can reach the **alarm** state in less than 50 transitions:

$$EF_{[\#T < 50]} \text{Alarm}$$

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- we can reach the **alarm** state in less than 50 transitions:

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- we can reach the **alarm** state in less than 50 “ticks”:

$$EF_{[\#\text{tick} < 50]} \text{Alarm}$$

Timed CTL over KSs with **tick**: $E\varphi U_{<k} \psi \stackrel{\text{def}}{=} EF_{[\#\text{tick} < k]} \psi$

Examples of formulas

- For an ATM: “it is not possible to get money when three mistakes are made in the same session”:

$$\text{AG}(\neg \text{EF}_{[\#error \geq 3 \wedge \#reset = 0]} \text{money})$$

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- $\text{AG}(\text{EF}_{[\#(\text{EXalarm}) \leq 5]} \text{init})$
“It is always possible to reach `init` along a path where less than 5 states have an `alarm` state as successor.”

Examples of formulas

- The bounded waiting property with bound 10 for a mutual exclusion algorithm with n processes:

$$AG \bigwedge_i (\text{request}_i \Rightarrow \neg EF_{[\sum_{j \neq i} \#CS_j > 10 \wedge \#CS_i = 0]} \top)$$

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- “The number of **receive** events can not exceed the number of **send** events”:

$$AG_{[\#send - \#receive < 0]} \perp$$

Examples of formulas

- **Quantitative fairness:** “the φ_i 's occur infinitely often along every run and there is no sub-run where φ_i holds for more than 10 states and φ_j holds for less than 4 states”:

$$\text{AG AF}_{[\wedge_i 5 \leq \# \varphi_i \leq 10]} \top$$

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$$\text{EF}_{[100 \cdot \# \text{error} - \# \top < 0]} P$$

(Constraints “ $\frac{\#P}{\#P'} \sim k$ ” can easily be expressed.)

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Constraints: $\bigwedge \bigvee \sum_{i=1}^l \alpha_i \cdot \#\varphi_i \sim k$

Proposition

CCTL with positive coefficients can be translated into CTL.

Idea: count manually occurrences of events using nested U modalities. . . and consider all possible shuffles of such occurrences.

Expressiveness

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Proposition

CCTL with positive coefficients can be translated into CTL.

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Proposition

The formula $\varphi = \text{AG}_{[\#A - \#B < 0]} \perp$ cannot be translated into CTL.

Idea: the set of models of any CTL formula can be recognized by a Büchi alternating tree automaton. For φ , we need a counter for any path.

Diagonal constraints increase the expressive power of CTL.

Succinctness

CCTL formulas with constraints $\bigwedge \bigvee \sum_{i=1}^l \alpha_i \cdot \#\varphi_i \sim k$ with **positive coefficients** can be translated into CTL, but in these constraints, there are **three potential sources of concision**:

- the binary encoding of constants,
- the Boolean combinations in the constraints,
- and the sums.

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- the **binary encoding of constants**,
- the **Boolean combinations in the constraints**,
- and the sums.

Only the two first yield an exponential improvement in succinctness.

Succinctness – binary encoding of the constants

The natural translation of $EF_{[\#A=k]}B$ into CTL yields an exponential formula.

(it uses k nested modalities...)

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The natural translation of $EF_{[\#A=k]}B$ into CTL yields an exponential formula.

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Proposition CCTL with constraints $\# \varphi \sim k$ can be exponentially more succinct than CTL.

Idea: TCTL formulas $EF_{<k}A$ and $EF_{>k}A$ do not admit any equivalent CTL formula of temporal height less than k [LST03].

Succinctness – Boolean combinations

Proposition CCTL with constraints $\bigwedge (\#\varphi_i \sim k_i)$ and unary encoding of integers can be exponentially more succinct than CTL.

Succinctness – Boolean combinations

Proposition CCTL with constraints $\bigwedge (\#\varphi_i \sim k_i)$ and unary encoding of integers can be exponentially more succinct than CTL.

Idea: any CTL formula equivalent to ψ :

$$\psi = E(F P_0 \wedge \dots \wedge F P_n)$$

must be of length exponential in n [Wil99, AI03].

$$\psi \equiv EF_{[\bigwedge_i \#P_i \geq 1]} \top$$

(no need of binary encoding of constants)

Succinctness – Sums

Proposition For every formula $\Phi \in \text{CCTL}$ with constraints of the form $\sum \#\varphi_i \sim k$ and with **unary encoding** of integers, there exists an equivalent CTL formula of **DAG-size** polynomial in $|\Phi|$.

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Succinctness – Sums

Proposition For every formula $\Phi \in \text{CCTL}$ with constraints of the form $\sum \#\varphi_i \sim k$ and with **unary encoding** of integers, there exists an equivalent CTL formula of **DAG-size** polynomial in $|\Phi|$.

Example: $\Phi = \text{EF}_{\sum_i \#P_i = K} A$ is equivalent to Ψ_K with:

$$\Psi_k \stackrel{\text{def}}{=} E \left(\bigwedge_i \bar{P}_i \right) U \left(\bigvee_i P_i \wedge \beta_{k,1,\perp} \right) \quad (k > 0)$$

$$\Psi_0 \stackrel{\text{def}}{=} E \left(\bigwedge_i \bar{P}_i \right) U A$$

$$\Psi_{-1} \stackrel{\text{def}}{=} \perp$$

$$\beta_{k,i,\epsilon} \stackrel{\text{def}}{=} (P_i \wedge \beta_{k-1,i+1,\top}) \vee (\bar{P}_i \wedge \beta_{k,i+1,\epsilon}) \quad (i < n)$$

$$\beta_{k,n,\top} \stackrel{\text{def}}{=} (P_n \wedge \text{EX } \Psi_{k-1}) \vee (\bar{P}_n \wedge \text{EX } \Psi_k)$$

$$\beta_{k,n,\top} \stackrel{\text{def}}{=} P_n \wedge \text{EX } \Psi_{k-1}$$

Comparison with Past

Counting constraints deal with past events !

We could use past-time modalities:

$$\text{AG}(\text{money} \Rightarrow \neg F_s^{-1}(\text{error} \wedge F_s^{-1}(\text{error} \wedge F_s^{-1}\text{error})))$$

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- + Past-time modalities allow us to express properties over the **ordering of the events**.
- + They (often) increase the expressive power (compared to CTL).
- + Boolean combinations are directly handled...
- Counting constraints are still more succinct.
- Complexity (model-checking $\text{CTL} + F^{-1}$ is PSPACE-Complete)

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Model checking algorithms

The complexity of the model checking problem depends on the type of the constraints.

Complexity ranges from P-complete to . . . undecidability.

Constraints

Given $l, k \in \mathbb{N}$, $k' \in \mathbb{Z}$ and $\sim \in \{<, \leq, =, \geq, >\}$, we define:

$$C_0: \#\varphi \sim k$$

$$C_1: \left(\sum_{i=1}^l \#\varphi_i \right) \sim k$$

$$\alpha C_1: \left(\sum_{i=1}^l \alpha_i \cdot \#\varphi_i \right) \sim k \quad \alpha_i \in \mathbb{N}$$

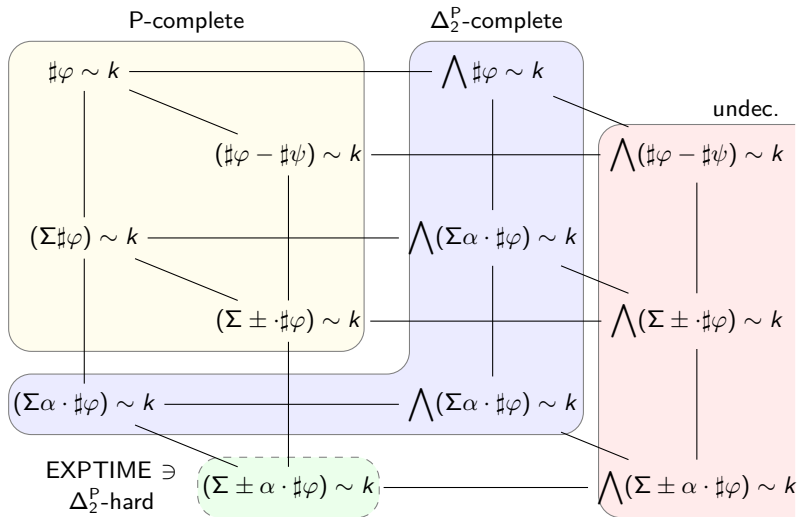
$$C_2: (\#\varphi - \#\psi) \sim k'$$

$$C_3: \left(\sum_{i=1}^l \pm \cdot \#\varphi_i \right) \sim k'$$

$$\alpha C_3: \left(\sum_{i=1}^l \beta_i \cdot \#\varphi_i \right) \sim k' \quad \beta_i \in \mathbb{Z}$$

And $B(C)$ is the set of **Boolean combinations** of constr. in C .

Conclusion



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Model checking $\text{CCTL}_{\mathcal{C}_1}$: $(\Sigma \# \varphi_i) \sim k$

Theorem Model-checking $\text{CCTL}_{\mathcal{C}_1}$ is P-complete.

Reduction to a model-checking problem for TCTL formula over Kripke structures with O/1 durations.

Model checking CCTL_{C_1} : $(\Sigma \# \varphi_i) \sim k$

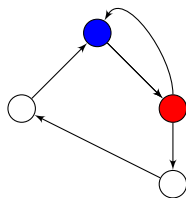
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Reduction to a model-checking problem for TCTL formula over Kripke structures with $O/1$ durations.

Example: $\mathcal{S} = (Q, R, \ell)$, and $\Phi = E\varphi U_{[\#P_1 + \#P_2 \sim k]} \psi$

● $\models P_1$

● $\models P_1 \wedge P_2$



Model checking CCTL_{C_1} : $(\Sigma \# \varphi_i) \sim k$

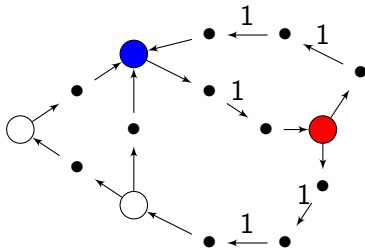
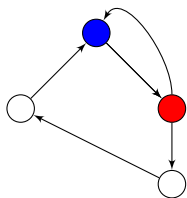
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● $\models P_1$

● $\models P_1 \wedge P_2$



$\Phi' = E(\neg \bullet \Rightarrow \varphi) U_{\sim k} (\neg \bullet \wedge \psi) \in \text{TCTL}$

Model-checking $\text{CCTL}_{\mathcal{C}_3}$

For $\text{CCTL}_{\mathcal{C}_3}$ (i.e. with constraint of the form " $(\sum_{i=1}^l \pm \#\varphi_i) \sim k$ "),
the previous encoding gives a DKS with durations in $\{-1, 0, 1\}$:

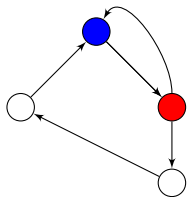
Model-checking CCTL_{C₃}

For CCTL_{C₃} (i.e. with constraint of the form " $(\sum_{i=1}^l \pm \cdot \# \varphi_i) \sim k$ "), the previous encoding gives a DKS with durations in $\{-1, 0, 1\}$:

$\mathcal{S} = (Q, R, \ell)$, and $\Phi = E\varphi U_{[\#P_1 - \#P_2 \sim k]} \psi$

● $\models P_1$

● $\models P_2$



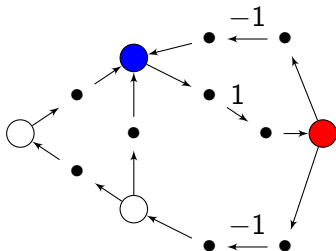
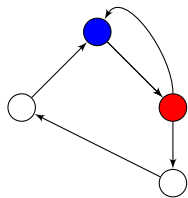
Model-checking $\text{CCTL}_{\mathcal{C}_3}$

For $\text{CCTL}_{\mathcal{C}_3}$ (i.e. with constraint of the form " $(\sum_{i=1}^l \pm \cdot \#\varphi_i) \sim k$ "), the previous encoding gives a DKS with durations in $\{-1, 0, 1\}$:

$\mathcal{S} = (Q, R, \ell)$, and $\Phi = E\varphi U_{[\#P_1 - \#P_2 \sim k]} \psi$

● $\models P_1$

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Reduction to a model-checking problem for TCTL over $\text{DKS}^{-1/0/1}$.

Model-checking TCTL over $\text{DKS}^{-1/0/1}$

Theorem

Model-checking TCTL over $\text{DKS}^{-1/0/1}$ is P-complete.

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- $\Phi \stackrel{\text{def}}{=} E\varphi U_{\leq k}\psi$: shortest paths + reachability of negative cycles.
- $\Phi \stackrel{\text{def}}{=} E\varphi U_{=k}\psi$:

Compute $R_k \stackrel{\text{def}}{=} \{(q, q') \in S^2 \mid \exists q \xrightarrow{k} q'\}$ as follows:

- $R_k = R_{\lfloor k/2 \rfloor} \cdot R_{\lfloor k/2 \rfloor} \cdot R_{k \% 2}$
- $R_1 \stackrel{\text{def}}{=} R_0 \cdot \xrightarrow{1} \cdot R_0$
- R_0 is the least solution of:

$$X = (\xrightarrow{0})^* \cup X \cdot (\xrightarrow{1} \cdot X \cdot \xrightarrow{-1} \cup \xrightarrow{-1} \cdot X \cdot \xrightarrow{1}) \cdot X$$

Add constraints for $\varphi \dots \rightsquigarrow R'_k$

$\Rightarrow q \models \Phi$ iff $(q, q') \in R'_k$ for some q' satisfying ψ' .

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- $\Phi \stackrel{\text{def}}{=} A\varphi U_{\sim k}\psi : \dots$

Model-checking $\text{CCTL}_{\mathcal{C}_3}$: $(\Sigma \pm \cdot \#\varphi_i) \sim k$

Corollary Model-checking $\text{CCTL}_{\mathcal{C}_3}$ is P-complete.

Outline

- 1 CCTL
- 2 Expressivity
- 3 Model checking**
 - Efficient model-checking
 - and harder. . .**
 - and even worst !
- 4 Linear-Time Temporal Logics
- 5 Conclusion

Model-checking $\text{CCTL}_{\mathcal{B}(C_0)}$: $\bigwedge \bigvee \# \varphi \sim k$

Theorem

Model-checking $\text{CCTL}_{\mathcal{B}(C_0)}$ is Δ_2^P -hard.

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Theorem

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Reduction from SNSAT:

$$\left\{ \begin{array}{l} z_1 \stackrel{\text{def}}{=} \exists X_1. \varphi_1(X_1) \\ z_2 \stackrel{\text{def}}{=} \exists X_2. \varphi_2(z_1, X_2) \\ \dots \\ z_n \stackrel{\text{def}}{=} \exists X_n. \varphi_n(z_1, \dots, z_{n-1}, X_n) \end{array} \right.$$

Question: $z_n = \top$?

Model-checking $\text{CCTL}_{\alpha\mathcal{C}_1}$: $(\sum_{i=1}^l \alpha_i \cdot \#\varphi_i) \sim k, \alpha_i \in \mathbb{N}$

Theorem

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Reduction from the MC pb for TCTL over **KSSs with durations**.

Let $\mathcal{S} = (Q, R_{\mathcal{S}}, \ell)$ be a DKS.

“ $q \xrightarrow{k} q'$ in \mathcal{S} ” is replaced by “ $q(\text{ok}) \rightarrow q_{aux}(P_k) \rightarrow q'(\text{ok})$ ”

The TCTL formula $\Phi \stackrel{\text{def}}{=} E\varphi U_{\sim m}\psi$ is replaced by $\tilde{\Phi}$:

$$E(\text{ok} \Rightarrow \tilde{\varphi}) U_{[C]} (\text{ok} \wedge \tilde{\psi})$$

with $C \stackrel{\text{def}}{=} \sum_{d \in W} d \cdot \#P_d \sim m$.

Model-checking $\text{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$: $\forall \bigwedge (\sum_{i=1}^l \alpha_i \cdot \#\varphi_i) \sim k,$
 $\alpha_i \in \mathbb{N}$

Theorem

Model-checking $\text{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$ is in Δ_2^P .

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Model-checking $\text{CCTL}_{\mathcal{B}(\alpha\mathcal{C}_1)}$ is in Δ_2^P .

Based on the Parikh image of the runs satisfying $\text{EF}_{[C]}\psi$.

- we can assume that $|\rho|$ is in $O(|Q| \cdot 2^{|C|})$;
- check in polynomial time that a guessed Parikh image corresponds to some path;
- verify that it verifies the formula.

And for $\text{EG}_{[C]}\psi$, we look for an infinite run but $(\sum \alpha_i \cdot \#\varphi_i) \sim k$ may change its truth value at most twice along the run...

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Model-checking $\text{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$: $\bigwedge V(\#\varphi - \#\psi) \sim k$

Theorem

The model-checking problem for $\text{CCTL}_{\mathcal{B}(\mathcal{C}_2)}$ is undecidable.

Model-checking $\text{CCTL}_{\mathcal{B}(C_2)}$: $\bigwedge V(\# \varphi - \# \psi) \sim k$

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The model-checking problem for $\text{CCTL}_{\mathcal{B}(C_2)}$ is undecidable.

Reduction from the halting problem of a two-counter machine \mathcal{M} .

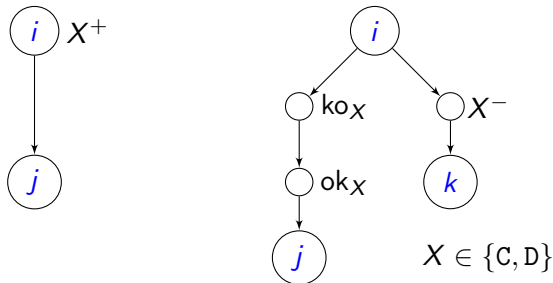
Two counters: C and D, and 5 kinds of instructions:

- i : C++, goto j
- i : D++, goto j
- i : if C=0 then goto j else C--, goto k
- i : if D=0 then goto j else D--, goto k
- i : halt

Model-checking $\text{CCTL}_{\mathcal{B}(C_2)} : \bigwedge V(\#\varphi - \#\psi) \sim k$

From \mathcal{M} , we build the following Kripke structure:

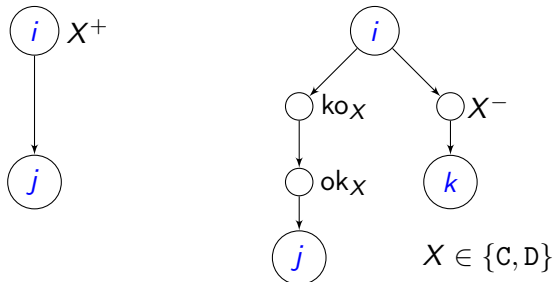
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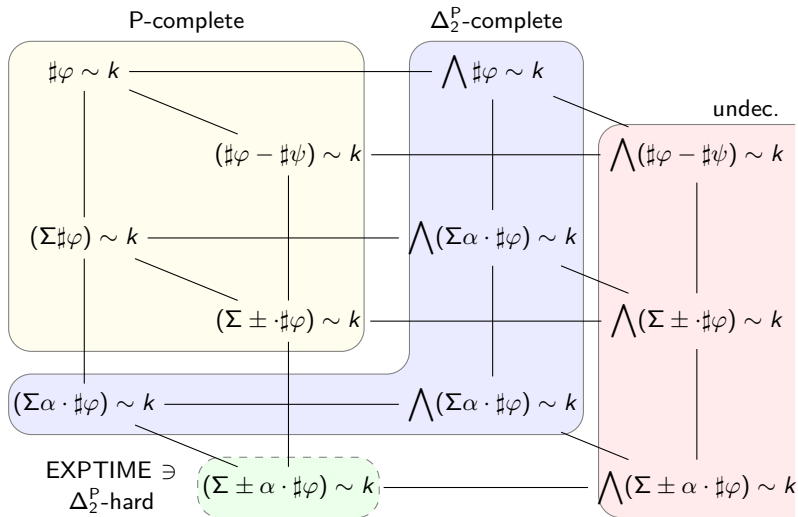


\mathcal{M} does not halt *if and only if* $q_1 \models_{S_{\mathcal{M}}} \text{EG}_{[C]} \perp$ with:

$$C \stackrel{\text{def}}{=} (\# \text{halt} \geq 1) \vee C_{\text{bad}}$$

$$C_{\text{bad}} \stackrel{\text{def}}{=} \bigvee_{X \in \{C, D\}} \left((\# X^+ - \# X^- < 0) \vee (\# X^+ - \# X^- > 0 \wedge \# \text{ko}_X - \# \text{ok}_X > 0) \right)$$

Conclusion



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LTL + counting constraints

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$$LTL \ni \varphi, \psi ::= P \mid \varphi \wedge \psi \mid \neg \varphi \mid \varphi U \psi \mid X \varphi$$

where $P \in AP$.

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$\rho \models P$ iff $P \in \ell(q_1)$

$\rho \models \varphi \wedge \psi$ iff $\rho \models \varphi$ and $\rho \models \psi$

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$\rho \models \varphi U \psi$	iff	$\exists k \geq 0, \rho^k \models \psi$, and $\forall 0 \leq i < k, \rho^i \models \varphi$
$\rho \models \varphi U_{[c]} \psi$	iff	$\exists k \geq 0, \rho^k \models \psi$, $\rho, k-1 \models C$ and $\forall 0 \leq i < k, \rho^i \models \varphi$

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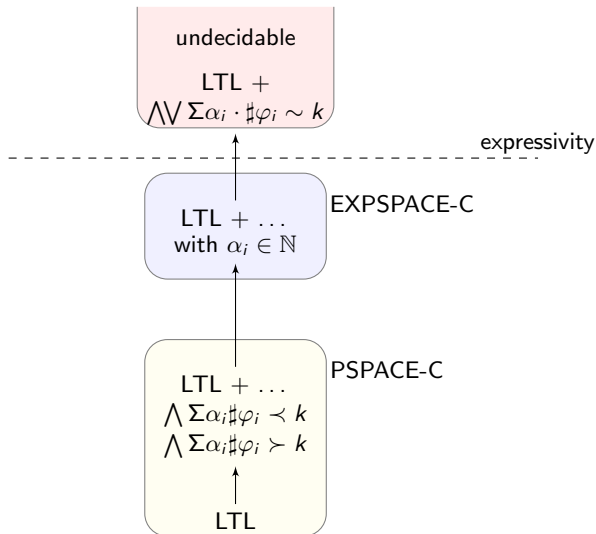
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iff

Overview of the results for LTL



Linear-time case

CLTL $\stackrel{\text{def}}{=} LTL+$ constraints with “positive” coefficients.

Use **standard** techniques:

- alternating Büchi automata to recognize the models of a formula.
(for CLTL, \mathcal{A}_ϕ is exponential in $|\Phi|$.)
- satisfiability and model-checking
- removing equality makes procedures more efficient.
- counting along a unique path is easier...

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Conclusion

- Other semantics are possible. . . For example, the **cumulative** semantics.

$EF_{[\#T \leq k_1]}^c(P_1 \wedge EF_{[\#T \leq k_2]}^c P_2)$: “there is a run with less than k_2 transitions leading to P_2 and along this run there is some P_1 located at less than k_1 transitions from the initial state”

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- See in practice.
- Consider other operators (modulo, . . .).
- Consider properties over infinite runs (ratio).



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