Progress (and Lack Thereof) for Graph Coloring Approximation Problems

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Abstract. We examine the known approximation algorithms for the classic graph coloring problem in general graphs, with the aim to extract and restate the core ideas. We also explore a recent edge-weighted generalization motivated by the modeling of interference in wireless networks. Besides examining the current state-of-the-art and the key open questions, we indicate how results for the classical coloring problem can be transferred to the approximation of general edge-weighted graphs.

1 Introduction

Graph coloring is among the most fundamental NP-hard optimization problems. It forms the basic model of conflict resolution, or scheduling with conflicts, and the allocation of scarce resources, such as access to spectrum in wireless computing.

Formally, given an input graph $G = (V, E)$ with vertex set V and edge set E, a proper (vertex) *coloring* is an assignment $\pi : V \rightarrow \{1, 2, \ldots, \}$ such that adjacent vertices receive different colors, i.e., $uv \in E$ implies that $\pi(u) \neq \pi(v)$. The objective of the *coloring problem* is to minimize the number of colors used, i.e., the largest value $\pi(v)$. The *chromatic number* $\chi(G)$ is the minimum number of colors used in a proper coloring of G. Let n denote $|V|$.

The aim of this note is to survey the state of affairs in the development of approximation algorithms for graph coloring. Namely, we focus on inexact algorithms that offer performance guarantees: the *performance (ratio)* of a coloring algorithm is largest ratio between the number of colors used by the algorithm on an instance G to the chromatic number of G. This ratio may be a function of some parameter of the graph, most commonly n, the number of vertices. We consider here general graphs, i.e., the class of all graphs, rather than (possibly) more manageable graph families.

We survey the known *polynomial-time* approximation algorithms for general graphs, with the aim to gather and restate the key ideas used in these algorithms. This is a study that extends back to early seventies, but curiously enough, the trail dries up by 1990. In the interim, research in lower bounds has brought extensive and illustrious advances, which may motivate renewed efforts on upper bounds.

We also introduce a recently proposed generalization of coloring involving edge-weighted graphs. This is motivated by scheduling in wireless networks.

2 Approximation Algorithms for Coloring General Graphs

It is useful to break the task of coloring the whole graph into *progress* steps. Informally, an operation yields *progress towards a T-coloring* if a repetition of such operations results in a valid T-coloring.

We first note that the coloring task can be reduced to the apparently easier task of finding a large *independent set*, i.e., a subset of mutally non-adjacent vertices. This set can be assigned a fresh color, and the task repeated until all the vertices have been colored. In general, such a reduction carries a log-factor overhead (as known from the approximation of the Set Cover problem). However, since we are content with performance ratios that are a *polynomial* in n when treating general graphs, the overhead factor actually reduces to a constant.

Example: Suppose we are given a $\sqrt{n}$-approximation algorithm for independent sets in χ-colorable graphs. We argue that repeated application results in a $4\sqrt{n}$-approximation for coloring. Namely, $\chi\sqrt{n}$ applications (using that many colors) reduce the number of vertices to at most $n/2$. Halving the number of vertices further takes at most another $\chi\sqrt{n/2}$ applications. Continuing, the number of colors used form a geometric sequence of $\chi\sqrt{n}(1+\sqrt{2}^{-1}+\sqrt{2}^{-2}+\cdots) \leq \chi\sqrt{n}/(1-1/\sqrt{2}) < 3.5\chi\sqrt{n}$, which yields a $3.5\sqrt{n}$-approximation.

Observation 1. *Repeated application of a ρ-approximation algorithm for independent sets in χ-colorable graphs results in a $O(\rho)$-approximation algorithm for coloring, whenever $\rho = \Omega(n^{\epsilon})$, for some $\epsilon > 0$.*

Johnson One simple progress step involves identifying a smallest degree vertex v. We add v to our independent set solution and recurse on the set $\overline{N[v]}$ of vertices non-adjacent to v.

Johnson [17] analyzed several graph coloring heuristics and showed that the *minimum-degree greedy* heuristic attained a non-trivial performance ratio. The heuristic can be viewed as a repeated application of the above progress step. His observation was that for a minimum degree vertex v, the number $\overline{N[v]}$ of non-neighbors is at least $n/\chi - 1$, since all nodes that belong to the largest color class A must be non-adjacent to the other nodes in the class, and A must contain at least n/χ vertices. By repeatedly selecting a minimum degree vertex and eliminating its neighbors, we obtain an independent set whose size $S(n)$, as a function of the number of nodes n in the graph, can be given by the recurrence relation $S(n) \geq 1 + S(n/\chi)$ and $S(1) = 1$. The solution of this recurrence is $S(n) = \log_{\chi} n$. It follows that

Observation 2. *Selecting the minimum-degree vertices makes progress towards a $O(n/\log_{\chi} n)$-coloring.*

The performance ratio of the minimum-degree algorithm is therefore at most $O(n/(\chi \cdot \log_{\chi} n))$. This is maximized when χ is constant, for a performance ratio of $O(n/\log n)$.

Wigderson Wigderson [22] observed that 3-colorable graphs could be colored with $O(\sqrt{n})$-colors. Namely, we can consider two cases depending on vertex degrees. If there is a vertex v of degree at least $\sqrt{n}$, then we can make progress as follows. We note that the graph $G[N(v)]$ induced by the neighbors of v is 2-colorable; thus by assigning it two fresh colors, we make progress towards a $O(\sqrt{n})$-coloring. On the other hand, if there is a vertex of degree less than $\sqrt{n}$, we can make progress towards a $O(\sqrt{n})$-coloring in the same way as argued for the min-degree heuristic.

By generalizing his argument, we can always make progress towards a $O(\chi n^{\chi-1})$-coloring by considering two cases depending on vertex degree: either there is a node of degree at most $n^{\chi-1}$, or there is a node v of higher degree in which case its sizable neighborhood is $\chi - 1$ colorable and by induction we make progress.

Observation 3. *We can always make progress towards a $O(\chi n^{\chi-1})$-coloring of χ-colorable graphs.*

If we now combine this bound with Johnson's, selecting the better of the two bounds, we obtain an algorithm with performance ratio at most $O(\min(n^{\chi-1}, n/(\chi \log_\chi n))$, which is easily seen to be $O(n(\log\log n/\log n)^2)$.

Berger and Rompel Even if there are no truly low-degree vertices, we may still find a *set* of independent vertices whose combined neighborhood is relatively small, allowing us to generalize the min-degree approach of making progress.

Consider the largest color class A, whose size is clearly at least n/χ. If we pick a subset S in A, then none of the other vertices $A \setminus S$ are adjacent to nodes in S, obviously. Thus, in particular, we can see that if we define $\overline{N(S)}$ to be the set of vertices non-adjacent to all nodes in S, we get that $|\overline{N(S)}| \geq n/\chi - |S|$. Since S will generally be much smaller than n/χ, finding such a set S allows us to make progress towards finding a $\Omega(|S| \cdot \log_\chi n)$-independent set, resulting in a $O(n/(|S| \log_\chi n))$-coloring.

But how to find such a set of non-trivial size? We have no certificate of what it means to be in the largest color class A. Here we are helped by abundance: A is large so it contains a lot of subsets. In particular:

Observation 4. *A random subset of size $K = \log_\chi n$ is contained in A with probability at least $1/n$.*

Thus, we can pick random subsets until we find one that satisfies the properties of belonging to A. Berger and Rompel [4] derandomized this argument to obtain a deterministic method with the same performance.

It follows that we can strengthen the min-degree approach considerably:

Observation 5. *One can always make progress towards a $O(n/(\log_\chi n)^2)$-coloring.*

If we combine this progress bound with Wigderson's, we obtain a coloring with performance ratio of $\min(\chi n^{\chi-1}, n/(\chi(\log_\chi n)^2))$. This is maximized when $\chi = \Theta(\log n/\log\log n)$, for a performance of $O(n(\log\log n/\log n)^3)$.

Note that the hardest cases for all these approximation algorithms are when $\chi(G)$ is $\Theta(\log n/\log\log n)$. Wigderson's approach then fails to deliver, leaving us with a $O(\log n/\log\log n)^2$-sized independent set.

Further improvement One may wonder if Observation 4 can be leveraged to get stronger properties on the subset S beyond the basic bound of n/χ on the non-neighborhood size. The key observation (from [9]) is that when the non-neighborhood size is small, nearly all of it must belong to the independent set A. But then, one can bring to bear algorithms to approximately find large independent sets. Specifically, the Ramsey method of Boppana and Halldórsson [6] achieves equivalent approximation factor for the independent set problem as Wigderson's method gave for the coloring problem.

We are thus led to the following strategy. We search for an independent set S of size $K = \log_\chi n$ that satisfies properties that hold for subsets of A. Then, one of two things must happen:

1. The Ramsey method finds a large independent set in $\overline{N(S)}$ (specifically of size $\Omega(\log^3 n)$), or
2. The non-neighborhood $\overline{N(S)}$ is larger than we previously argued.

Specifically, in the latter case the non-neighborhood must be of size $\Omega(\log_\chi n \cdot \log n/\log(\chi \log\log n/\log n))$, which in the range of interest for χ is $\Omega(\log_\chi n \cdot \log n)$.

When combined with Wigderson's bound, we obtain a performance ratio of $O(n(\log\log n)^2/\log^3 n)$, shaving off a loglog-factor.

Additional results Progress on lower bounds on approximability, based on the PCP theory [2], has been extensive since the early nineties. Most of the work is on the somewhat easier independent set problem, while Feige and Kilian [7] showed how to extend some of the results to chromatic number, in particular giving $n^{1-\epsilon}$-hardness, for any $\epsilon > 0$. The strongest hardness to date for independent sets is $n/2^{(\log n)^{3/4+\gamma}}$, for any $\gamma > 0$, by Khot and Ponnuswami [20].

The most promising direction for improved approximation algorithms for independent sets and coloring has for long been *semi-definite programming* (SDP), such as the θ-function of Lovász [21] and various hierarchies and strengthenings. For many families of problems, the best possible results achievable in polynomial time are obtained by SDPs. SDPs have been useful for coloring graphs of low chromatic number, including the best approximation known for 3-coloring of $\tilde{O}(n^{3/14})$ [5]. For general graphs, however, all the results on SDP for independent sets or coloring have been negative, with stronger lower bounds than the known inapproximability bounds.

We are led to ask what may possibly help algorithms for coloring general graphs. It is curious that it is almost a quarter of a century since the appearance of the last improved approximation results ([8] in 1990). The bold conjecture in [9] that the best possible approximability is only $\Theta(n/polylog(n))$ may still be validated. That still leaves some room for improvement.

3 Coloring Edge-Weighted Graphs

We address a generalization of the classic graph coloring problem. A coloring can be viewed as a partition of the vertices into sets of ultimate sparsity: each vertex has degree *less than 1* (i.e., zero) from the other nodes in the same color class. We assume the same constraint, with the only difference that degrees are now weighted. In fact, the graph need not be symmetric, so we have an edge-weighted digraph. Thus, we now seek a partition into fewest possible vertex subsets such that each node has weighted in-degree less than 1 from other nodes in the same set.

Our motivation for this problem comes from modeling interference in wireless networks. Whereas the classical TCS approach is to model interference as a pairwise binary property (i.e., one that can be captured by a graph), a more refined view commonly used by engineering communities is that interference is a many-to-one relationship. Having a conversation in a room where another discussion is going on may work fine, but once the room becomes crowded with speakers, listening becomes progressively harder. In other words, it is the *cumulative* effect of multiple transmitters that matters when assessing whether a message can be properly decoded.

Formally, we are given a digraph $H = (V, E)$ with non-negative weights $w : E \to \mathbb{R}^+$ on the edges. A subset S of vertices is *independent* if the in-degree within S of each node is strictly less than 1, i.e., if $\sum_{u \in S} w(u, v) < 1$, for each $v \in S$. A coloring is, as before, a partition into independent sets, and we aim to use the fewest colors possible. Notice that in the case of symmetric 0/1-weights, the definition coincides with classical graph coloring.

We will first focus on coloring approximation in terms of the parameters n and χ, before examining the sparser instances that relate to the specific wireless applications.

3.1 Coloring general (edge-weighted) graphs

The question of how well we can handle general edge-weighted graphs is interesting from a basic science standpoint, even if the results are too weak for most applications.

A very simple approach yields an easy $O(n/\log n)$-approximation (proposed for the wireless scheduling problem in [16], while the generic approach was perhaps first stated in [10]): Partition the graph into $n/\log n$ vertex-disjoint sets, and color each set optimally with a fresh set of colors. Since such an optimal coloring of a graph on n vertices can be obtained in time about 3^n [16], using the technique of inclusion-exclusion, the time complexity of coloring the $\log n$-sized sets is polynomial.

The only other reported result involves the case of graphs that contain an independent set of size $(1 + \epsilon)n/2$, for some $\epsilon > 0$. In this case, a semi-definite programming formulation results in an algorithm to produce an independent set of size $\Omega(\epsilon n)$ [13]. This is, however, insufficient to provide non-trivial bounds even for 2-colorable graphs.

We propose here different approaches that emulate some of the results obtained for ordinary graphs. We skip Johnson's approach and start with Berger and Rompel's. Recall that $\chi = \chi(H)$ is the chromatic number of the edge-weighted graph H.

Proposition 1. *There is an algorithm that finds a $O(n/(\log_{\chi \log n} n)^2)$-coloring of an edge weighted graph H.*

Let k be such that $k = \chi(H) \cdot \log_k^2 n$, and note that $\log k = \Theta(\log(\chi \log n))$. Let $X = \log_k^2 n$. We form a (classical) graph G on the same vertex set as H, where $uv \in E(G)$ iff $w(u, v) + w(v, u) \geq 1/X$. We then apply the algorithm of Berger and Rompel on G to obtain an independent set I in G, but retain at most X of the nodes in I. This set is then a feasible (edge-weighted) independent set in H, since all weights of edges within I are less than $1/X$.

It remains to argue a lower bound on the size of the independent set I found. Consider a color class C in H and a node v in C. Observe that v has fewer than X neighbors within C in G, thus C induces a subgraph in G of maximum degree less than X. It follows that $\chi(G) \leq X \cdot \chi(H) = k$. The Berger-Rompel algorithm produces a set I of size $\Omega(\log^2_{\chi(G)} n) = \Omega(X)$. Hence, we make progress towards a $O(n/X)$-coloring, as claimed.

We next turn to Wigderson's approach. We start with the case of 2-colorable graphs, the first case that is not computationally easy in the edge-weighted setting. With hindsight, we actually aim to handle a more general case. A graph H is said to be *t-almost k-colorable*, for parameters t and k, if there is a subset of $tn^{1/k}$ vertices whose removal leaves the graph k-colorable.

Proposition 2. *There is an algorithm that finds a $O(t\sqrt{n})$-coloring of t-almost 2-colorable edge-weighted graphs.*

Set $X = \sqrt{n}$ and form again the graph $G = G_X$ defined as before. Again, we see that an independent set in G of size at most X is independent in H. If there is a vertex of degree at most $3(t + 1)n/X$ in G, we can clearly make progress towards a $O(tX)$-coloring, as desired.

On the other hand, consider a vertex v of degree at least $3(t + 1)n/X$ and let N_v denote its set of neighbors. Our aim is to apply the SDP approach on the induced subgraph $H[N_v]$, and for that purpose we need to show that its independence number is high. Let C denote the set of (at most tX) vertices whose removal makes H to be 2-colorable, and let A and B denote the two color classes. Suppose v is in A, without loss of generality, and let $N_A = N_v \cap A$ be the subset of N_v from A. Note that N_A is of size at most X (since the sum of the incoming weights from nodes in A is less than 1 and each such edge weight is at least $1/X$). The set $N_v - N_A - C$ must be a subset of B, and its fraction of N_v is at least

$$\frac{|N_v - N_A - C|}{|N_v|} \geq 1 - \frac{|X| + |tX|}{|N_v|} \geq 1 - \frac{1 + t}{3(t + 1)} = \frac{2}{3} .$$

Thus, the induced subgraph $H[N_v]$ contains a feasible subset of at least two thirds of the nodes. We can then apply the SDP result of [13] with $\epsilon = 1/3$ to obtain an independent set I of size $\Omega(\epsilon|N_v|) = \Omega(tX)$, making progress towards a $O(n/(tX)) = O(\sqrt{n}/t)$-coloring, as desired.

We now apply the approach recursively, along the lines of Wigderson.

Proposition 3. *There is an algorithm that finds an independent set of size $\Omega(n^{1/k}/(t+k))$ in a t-almost k-colorable edge-weighted graphs. Thus, we can make progress towards a $O((t+k)n^{1-1/k})$-coloring.*

When $k = 2$, the claim holds from Prop. 2, so assume $k \geq 3$. Let $X = n^{1/k}$ and form $G = G_X$ as before. If there is a vertex of degree at most n/X in G, then by selecting it we make progress towards a $O(n/X) = O(n^{1-1/k})$ coloring. Otherwise, consider the set N_v of neighbors of an arbitrary vertex v, which is of size greater than $n/X = n^{1-1/k}$. We claim that $H[N_v]$ is $(t+1)$-almost $k-1$-colorable. Namely, N_v might contain the at most tX nodes whose removal turns H into a k-colorable graph H', and it could contain at most X nodes that belong to the same color class in H' as v, but together this amounts to only $(t+1)n^{1/k} \leq (t+1)(|N_v|^{k/(k-1)})^{1/k} = (t+1)|N_v|^{1/(k-1)}$, which satisfies $(t+1)$-almost $(k-1)$-colorability. By applying our method by induction on $H[N_v]$, we obtain an independent set I of size at least

$$c\frac{|N_v|^{1/(k-1)}}{(t+1)+(k-1)} = c\frac{|N_v|^{1/(k-1)}}{t+k} \geq c\frac{n^{1/k}}{t+k} \ ,$$

for some absolute constant c, thus making progress towards a $(t+k)n^{1-1/k}$-coloring, as desired.

In particular, we obtain a $O(kn^{1-1/k})$-coloring of k-colorable graphs.

Finally, we can combine the two approaches (Props. 1 and 3) to get approximation results for coloring general edge-weighted graphs that almost matches the best ratio known for ordinary graphs. The maximum of the two bounds is achieved when $\chi = \theta(\log n/\log\log n)$.

Corollary 1. *There is a $O(n(\log\log n/\log n)^3)$-approximate algorithm for coloring edge-weighted graphs.*

3.2 Better solvable cases

The instances that arise in wireless settings are not completely arbitrary. They have structure that distinguish them from general instances. Let us examine some of the structural properties that help in getting better solutions.

Wireless transmissions take place in physical space, and interferences generally speaking decreases with distance. One may expect there to be limits to how much a single transmission can be disturbed. Graph theoretically, it is natural to consider graphs of *low maximum in-degree* $\Delta^-(H)$. It was shown in [14] that one can color such graphs using $\lfloor 2\Delta^- + 1 \rfloor^2$ colors. Very recently, an optimal upper bound of $\lfloor 2\Delta^- + 1 \rfloor$ colors was obtained [3], which can be made constructive up to an arbitrarily small error term. A tight bound of $\lceil \Delta + 1 \rceil$ was also recently given for undirected edge-weighted graphs [1]. That resolves these questions. However, there is no clear link between maximum degree and the chromatic number, thus it does not directly address the question of efficient approximability.

Another parameter of sparsity that connects well with classical colorings is inductiveness. A graph is *ρ-inductive* if there is an ordering of the vertices so that for each vertex v, the in-degree of v from the nodes succeeding it in the order is at most ρ. Hoefer et al. [15] were among the first to treat wireless scheduling problems as edge-weighted graphs. They also introduced measures related to inductiveness in the study of the corresponding independent set problem. Earlier, Kesselheim and Vöcking [19] had actually shown that for wireless instances (with certain regimes of fixed power assignments to the transmitters), $\rho(H) = O(\log n \chi(H))$. This leaves the question of how good colorings can be obtained in terms of the inductiveness parameter ρ.

Kesselheim and Vöcking [19] gave a *distributed* algorithm that uses at most $O(\rho \log n)$ colors, resulting in a $O(\log^2 n)$-approximation. The bound on ρ was later tightened to the optimal $\rho(H) = O(\chi)$ [11], resulting in a $O(\log n)$-approximation for coloring. However, this turns out to be best possible in terms of inductiveness alone; namely, there are instances such that $\chi(H) = \Omega(\rho(H) \log n)$ [3]. Thus, better-than-logarithmic solutions will need to take additional properties of wireless instance into account.

Constant factor approximations are known for the independent set problem of edge-weighted graphs for instances derived by wireless settings. Specifically, it holds for the main two variants based on the choice of power assigned to the senders: power depends only on the intended transmission distance [12], or the power can be arbitrary [18]. This immediately implies logarithmic approximations for the corresponding coloring problems. These results utilize the fact that the transmission take place between units embedded in a *metric* space. Each node of the graph corresponds to a communication *link*, a sender-receiver pair located in the metric, while the weight of an edge represents the disturbance that one transmission has on another transmission link. In particular, in the setting where all senders use the same power, the weight of the directed edge (u, v) from link (s_u, r_u) to link (s_v, r_v) is proportional to $(d(s_v, r_v)/d(s_u, r_v))^\alpha$, where α is a fixed positive constant.

The key question is whether the metric property can be brought to bear to obtain better approximation ratios than logarithmic. The holy grail would be to give an absolute constant factor approximation. So far, all attempts have failed. Still, there are no obvious *a priori* reasons why this cannot succeed.

Additionally, it would be interesting to characterize other classes of instances that admit efficient approximations.

4 Conclusions

We have explored two graph coloring problems: The classical one and a recent edge-weighted variation motivated by wireless applications. Much remains to be done to deepen our understanding, even for the classical version.

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