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Title: On the number of zeros of multiplicity r

Abstract: Let $F \in \mathbf{F}[X_1, \dots, X_m] \setminus \{0\}$ be a multivariate polynomial where \mathbf{F} is any field. The multiplicity of a zero of a multivariate polynomial can be defined; a generalization of the multiplicity of a zero of a univariate polynomial.

One can upper bound the number of zeros counted with multiplicity that F can have over S^m for some finite $S \subseteq \mathbf{F}$ using the generalized Schwartz-Zippel bound; it was suggested by Augot et al. [1] and was recently proved by Dvir et al. [2]. The bound is in terms of the total degree of F .

We present upper and lower bounds in terms of the exponents of the leading monomial (with respect to any lexicographic ordering). The upper bound is an improvement to the generalized Schwartz-Zippel bound. Further, we allow more general point ensembles, namely $S_1 \times \dots \times S_m \subseteq \mathbf{F}^m$ for finite S_i .

The bounds can be hard to calculate but quite some concrete bounds can be found on <http://zeros.spag.dk>.

We present closed formulas for some cases, among others, the cases where we give sufficient conditions for when the lower bound equals the upper bound.

See [3] or <http://zeros.spag.dk> for further information.

[1] Augot, D., Stepanov, M.: "Interpolation based decoding of Reed-Muller Codes", slides from talk at Special Semester on Grbner Bases and Related Methods, RICAM, 2006,

<http://www.ricam.oeaw.ac.at/specsem/srs/groeb/download/Augot.pdf>.

[2] Dvir, Z., Kopparty, S., Saraf, S., Sudan, M.: "Extensions to the Method of Multiplicities, with applications to Kakeya Sets and Mergers", arXiv:0901.2529v2, 2009, 26 pages.

[3] Geil, O., Thomsen, C.: "On the number of zeros of multiplicity r ", arXiv:0912.1436, 2009, 21 pages.