

# Abstracts for 10th Nordic Combinatorial Conference (NORCOM 2010)

Mathematics Institute, School of Computer Science  
Reykjavik University  
May 26–28, 2010

## 1 Invited talks

**Speaker:** Magnús M. Halldórsson

**Affiliation:** Reykjavik University

**Title:** How well do graphs capture wireless interference?

**Abstract:** Graphs are fundamental combinatorial objects that capture pairwise relationships between items. They have commonly been used for modeling incompatibilities in network communications.

A key issue in wireless communication is managing interference between simultaneous transmissions. Various formulations have been used to capture this. One model is to deem two transmitters incompatible if their spatial separation is less than a fixed cutoff. This leads to the class of *unit disk graphs* (UDG), for which a large literature now exists. Secondary interference can be caused by two senders simultaneously affecting the same receiver. This has led to the study of, e.g., distance-2 constrained colorings in graphs as well various other classes of graphs generalizing UDG's.

In comparison, the models used by electrical and communication engineers are of quite different nature. The most common is the *signal-to-noise-plus-interference* (SINR) model. This is considered to capture reality more closely, and is in fact also called the physical model. In this model, interference is considered to be *cumulative*, rather than binary, and an *all-to-all* relationship, rather than purely local. As such, it has various curious properties that differ significantly from graph-based models.

The perceived complexity of the model is the main reason why results on mathematical and algorithmic analysis within this model are only recently starting to appear. Yet, as we gain further understanding, the question remains if wireless interference is now permanently lost beyond the grasp of combinatorics. In other words, to which extent can we use graphs in the analysis of wireless interference?

We outline in this talk the essential features of the physical model of wireless interference, and discuss two important scenarios for which graphs capture very well the essential constraints involving the interference.

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**Speaker:** Juhani Karhumki

**Affiliation:** University of Turku

**Title:** Solution chains and independent systems of word equations

**Abstract:** Two fundamental properties of word equations are the following. First the defect theorem which claims that if a set of  $n$  words satisfies a nontrivial equation the elements of it can be expressed as products of at most  $n - 1$  words. Second the compactness property of word equations which states that any system of constant-free equations with a finite number of variable is equivalent to one of its finite subsystems. Of course, the equivalence means here that the systems have exactly the same solutions.

Both of these results state something which can be viewed as a dimension property of word equations. The goal of this presentation is to analyze in more details these phenomena, and in particular to point out that many natural and simply formulated problems are still un- answered.

Central notions of our considerations are independent systems of equations and chains of solution sets of word equations. We say that a system of (word) equations is independent if it is not equivalent to any of its proper subsystems. By a chain of solution sets of word equations we mean a strictly decreasing sequence of solutions of systems of equations where the previous system is always a proper subset of the next one. More intuitively the latter notion aims to capture how many constrains on words we can introduce in such a way that in each step we obtain a proper restriction.

It follows from the above mentioned compactness property that all sets of independent word equations, as well as chains of solutions sets are finite. However, and this is a major open problem of the field, no upper bound, depending e.g. on the number of unknowns, is known, and, in fact, it is not

known whether such a bound exists. The best lower bounds are quartic in terms of the number of unknowns, as we shall explain in this lecture.

In the above spirit we can also ask how many constraints (as equations) we can introduce such that they force only a minimal defect effect (guaranteed already by a single equation) or they just avoid the maximal defect effect (allowing still a non-cyclic solution set). We recall known results of this research, but even more interestingly again point out many open problems. Indeed, it is amazing that even in the case of three unknowns the basic problems are not solved. Namely, no upper bound is known for the maximal size of independent systems of three unknown equations, and accordingly no upper bound is known for maximal chains of such solution sets, either. The best known lower bound for such a maximal chain is 7.

This is joint work with Aleksi Saarela.

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**Speaker:** Christian Krattenthaler

**Affiliation:** University of Vienna

**Title:** Non-crossing partitions for reflection groups

**Abstract:** The past few years have seen the emergence of several new combinatorial objects associated to reflection groups. A rich theory has been developed up to now, with still many problems and conjectures remaining unresolved. In this talk, I shall mainly concentrate on the non-crossing partitions associated to (real and complex) reflection groups, as defined by Armstrong, Bessis and Reiner, Brady and Watt. I shall start by giving a short introduction in the basics of reflection groups, and subsequently I shall provide a survey of the many beautiful and often surprising results about the enumeration, the poset structure, its topology, etc., of these non-crossing partitions that have been obtained up to this date.

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**Speaker:** Nik Ruskuc

**Affiliation:** University of St Andrews

**Title:** Substitution decompositions and pattern classes of permutations

**Abstract:** Substitution (or, as they are sometimes known, modular or wreath) decompositions are a ubiquitous tool in many branches of combinatorics: graphs, tournaments, posets, etc. In this lecture I will lay down the foundations for the theory of substitution decompositions for pattern classes of permutations, and will then describe some initial work on this subject done jointly with Mike Atkinson and Rebecca Smith.

If  $\sigma = s_1 \dots s_n$  is a permutation of length  $n$ , and if  $\alpha_1, \dots, \alpha_n$  are arbitrary permutations, the *substitution* of  $\alpha_1, \dots, \alpha_n$  into  $\sigma$  is the permutation  $\tau = \sigma[\alpha_1, \dots, \alpha_n]$  consists of  $n$  consecutive subsequences isomorphic to  $\alpha_1, \dots, \alpha_n$  respectively, whose relative order is the same as the relative order of the terms of  $\sigma$ . For example  $231[21, 321, 12] = 4376512$ . A permutation which does not admit a (non-trivial) such decomposition is said to be *simple*.

A set of permutations  $X$  is said to be a *pattern class* if it is closed under the *involvement*, or, equivalently, if it can be defined as the set of all permutations *avoiding* a certain set  $B = B(X)$  of permutations (called the *basis* of  $X$ ). The *substitution closure* of a pattern class  $X$  is the class

$$\mathcal{C}(X) = \{\sigma[\alpha_1, \dots, \alpha_n] : \sigma, \alpha_1, \dots, \alpha_n \in X, |\sigma| = n\}.$$

We say that  $X$  is *closed* if  $X = \mathcal{C}(X)$ . This happens if and only if the basis of  $X$  consists entirely of simple permutations.

The following questions naturally arise: *Given a pattern class  $X$  with basis  $B$  what is the basis for  $\mathcal{C}(X)$ ? Under which conditions is it finite?* It is relatively easy to find a general answer to the first question: The basis of  $\mathcal{C}(X)$  consists of all simple permutations minimal subject to not belonging to  $X$ . But this leaves the second question wide open. I will describe a complete solution to this question in the case where  $X$  is *principal*, i.e. its basis is a singleton. It turns out that even in this special case the demarcation line between finitely based and non-finitely based closures is remarkably involved. To give a taster: If  $X$  is the class with basis 1234 then  $\mathcal{C}(X)$  is non-finitely based, while if  $Y$  has the basis 3412 then the basis of  $\mathcal{C}(Y)$  is finite. I will also discuss some obstacles and possible ways forward for resolving the full question.

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**Speaker:** Luca Q. Zamboni

**Affiliation:** Universit Lyon 1/Reykjavik University

**Title:** A generalization of the Euclidean algorithm generated by interval

exchange transformations

**Abstract:** We define a multi-dimensional generalization of the Euclidean algorithm arising from a class of dynamical systems known as interval exchange transformations. In this context, the usual sequence of partial quotients is replaced by an infinite path on a finite graph whose vertices are certain trees we call trees of relations. We will discuss the combinatorics behind this algorithm and present some links to other areas including polygonal triangulation, actions of the Temperley-Lieb algebras, and the secondary structure of RNA.

## 2 Contributed talks

**Speaker:** R.E.L. Aldred

**Affiliation:** University of Otago

**Title:** Edge proximity and matching extension in planar graphs

**Abstract:** A graph  $G$  is said to have property  $E(m, n)$  if it contains a perfect matching and for every pair of disjoint matchings  $M$  and  $N$  in  $G$  with  $|M| = m$  and  $|N| = n$ , there is a perfect matching  $F$  in  $G$  such that  $M \subseteq F$  and  $N \cap F = \emptyset$ . It is well known that no planar graph is  $E(3, 0)$  or even  $E(2, 1)$ . If we consider even planar triangulations and demand that the edges are suitably far apart we can improve on these results. In this talk we will look at some of these improvements and consider the properties  $E(m, n)$  for planar triangulations when various distance restrictions are imposed on the edges to be included and avoided in the extension.

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**Speaker:** Jörgen Backelin

**Affiliation:** Stockholms universitet

**Title:** Multinomial expressions summation asymptotic approximations

**Abstract:** In the study of probabilistic tournaments, questions arise about the asymptotic behavior of the following type:

Consider a (simple and undirected) graph  $G = ([r], E)$  (for some positive

integer  $r$ ). Define a quadratic form  $f(x_1, \dots, x_r)$  as the sum of the monomials  $x_j x_l$  ( $1 \leq j < l \leq r$ ), such that  $\{j, l\} \in E$ , and for every positive integer  $n$  form the multinomial coefficient sum

$$S(n) = \sum_{i_1, \dots, i_r} \binom{n}{i_1, \dots, i_r} \frac{1^{-f(i_1, \dots, i_r)}}{2}$$

(sum over non-negative integers  $i_1, \dots, i_r$ , such that the multinomial coefficients are defined). The answer turns out to be that

$$S(n) \sim C \cdot \alpha(G)^n$$

where  $\alpha(G)$  is the independence number of  $G$ , and  $C$  is the number of independent sets of size  $\alpha(G)$  in  $G$ .

I shall present somewhat more general, and also slightly more precise results. The methods are elementary and purely combinatoric. (However, the problem seems to be related to Laplace approximations, relevant for e.g. the analytic function  $\sum S(n)x^n$ .)

**Speakers:** Lubomira Balkova and Stepan Starosta

**Affiliation:** Czech Technical University in Prague

**Title:** Generalizations of Sturmian words

**Abstract:** Sturmian words - the infinite aperiodic words with the lowest possible complexity - have been studied extensively since their discovery in the 40s and many equivalent definitions have been found out since. In this talk, we summarize generalizations of combinatorial definitions to larger alphabets and reveal relations between them. Two generalizations of the characterizations of Sturmian words based on palindromes proposed in 1999 by Droubay and Pirillo are in particular in the center of our interest. We present new results concerning such generalizations.

**Speaker:** Anders Claesson

**Affiliation:** Reykjavik University

**Title:** Permutations in new guises

**Abstract:** Bousquet-Mlou et al. [arXiv:0806.0666] gave bijections between four classes of combinatorial objects, thus proving that they are equinumerous: certain matchings due to Stoimenow; unlabeled  $(2+2)$ -free posets; permutations avoiding a specific pattern; and so called ascent sequences. Inspired by their work we define a natural superset of Stoimenow's matchings whose cardinality is shown to be  $n!$ . Moreover, we define a set of  $(2+2)$ -free posets, also of cardinality  $n!$ .

This is joint work with Svante Linusson (KTH - Royal Institute of Technology).

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**Speaker:** Lars Eirik Danielsen

**Affiliation:** University of Bergen

**Title:** On graphs preserved by edge local complementation

**Abstract:** Orbits of graphs under the operations local complementation (LC) and edge local complementation (ELC), the latter also known as pivoting, appear in several different contexts. These graph operations have applications in topics such as isotropic systems, circle graphs, rank-width, interlace polynomials, and quantum states for use in quantum computation. We introduce a new class of graphs, called ELC-preserved graphs. A graph is ELC-preserved if it has an ELC orbit of size one, up to isomorphism. Through an exhaustive search, we find all ELC-preserved graphs of order up to 12 and all ELC-preserved bipartite graphs of order up to 14. We provide four general recursive constructions for infinite families of ELC-preserved graphs, called star expansion, clique expansion, Hamming expansion, and Hamming clique expansion. We show that all known ELC-preserved graphs, obtained from our search, arise from these constructions. We also prove that certain pairs of ELC-preserved graphs are LC equivalent. Binary error-correcting codes can be represented as ELC orbits of bipartite graphs, and ELC has been used to improve the performance of iterative decoding algorithms. We introduce ELC-preserved codes as codes corresponding to ELC-preserved graphs, and describe the properties of ELC-preserved codes obtained from our constructions.

**Speaker:** Robson da Silva

**Affiliation:** ICE-UNIFEI

**Title:** A Franklin-like proof for an identity

**Abstract:** We present a combinatorial proof for an identity involving the triangular numbers. The proof resembles Franklin's proof of Euler's Pentagonal Number Theorem.

This is joint work with J. Plinio Santos.

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**Speaker:** Niel de Beaudrap

**Affiliation:** University of Potsdam

**Title:** On a class of restricted unitary Cayley graphs

**Abstract:** We define "quadratic unitary Cayley" graphs  $G_n$ , whose vertex set is the ring of integers modulo  $n$ , and where residues  $a, b \pmod{n}$  are adjacent if and only if their difference is a quadratic residue. We consider the conditions under which these graphs decompose as a tensor product. As well, despite the fact that these graphs often fail to be tensor products, we may use tensor-based arguments to characterize the diameters of these graphs in terms of their prime-power factorization, as well as characterize the conditions on  $n$  for  $G_n$  to be a perfect graph.

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**Speaker:** Dejan Delic

**Affiliation:** Ryerson University

**Title:** Homomorphism Testing Strategies For Oriented Trees

**Abstract:** Given a finite digraph  $G$ , the problem  $\text{HOM}(G)$  which decides whether, given some finite digraph  $H$ ,  $H$  admits a homomorphism to  $G$ , is a problem conjectured to split the class of all finite digraphs into two subclasses: those  $G$  for which the problem is tractable (i.e. in  $P$ ) and those for which it is NP-hard. In order to prove that  $\text{HOM}(G)$  is tractable for some

fixed  $G$ , one needs to exhibit a polynomial algorithm which will terminate successfully for every digraph  $H$  for which such a homomorphism exists. All known examples of oriented trees for which the problem is provably tractable are such that a particular class of algorithms, the so-called consistency checks, demonstrate its membership in the class P. In this talk, we will show that all finite oriented trees for which the problem is tractable are of bounded width, i.e. the consistency checks are the adequate class of algorithms proving the tractability of  $\text{HOM}(T)$ , where  $T$  is a finite oriented tree.

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**Speaker:** Niklas Eriksen

**Affiliation:** Göteborg University/Chalmers University of Technology

**Title:** Great expectations—gene order rearrangement made (sort of) easy

**Abstract:** Evolutionary relationships can be derived by studying the order in which the genes appear. Common gene order changing operations in nature are the reversal and the block transposition. A relatively simple method to estimate the evolutionary distance between two species is to compute the minimal number of reversals or block transpositions needed to transform the gene order of one of the species into the second species' gene order. This number is called the edit distance and it gives a lower limit for the evolutionary distance that is reliable for closely related species but unreliable for distantly related species. Better estimates can be obtained by means of the expected edit distance between two genomes separated by  $t$  operations, a function of  $t$  that is hard significantly harder to compute than the corresponding edit distances.

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**Speaker:** Hilmar Haukur Gudmundsson

**Affiliation:** Reykjavik University

**Title:** Dyck paths, standard Young tableaux, and pattern avoiding permutations

**Abstract:** In this talk we present a generating function and a closed counting formula in two variables that enumerate a family of classes of permutations that avoid or contain an increasing pattern of length three and have a

prescribed number of occurrences of another pattern of length three. This gives a refinement of some previously studied statistics, most notably one by Noonan. The formula is also shown to enumerate a family of classes of Dyck paths and Standard Young Tableaux, and a bijection is given between the corresponding classes of these two families of objects. Finally, the results obtained are used to solve an optimization problem for a certain card game.

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**Speaker:** Harri Haanpää

**Affiliation:** Aalto University

**Title:** Counting Hamiltonian Cycles in Bipartite Graphs

**Abstract:** A computational method for counting Hamiltonian cycles in bipartite graphs is presented. The algorithm constructs subpaths of the Hamiltonian cycle in a subgraph induced by vertices within some maximum distance of a chosen starting vertex; by repeatedly incrementing the maximum distance, a dynamic programming algorithm results. For speedup, symmetries of the graph being examined are useful, and one need not consider the internal structure of the subpaths.

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**Speaker:** Freyja Hreinsdttir

**Affiliation:** University of Iceland

**Title:** Binomial Edge Ideals

**Abstract:** We introduce *binomial edge ideals* attached to a simple graph  $G$  and study their algebraic properties.

**Definition:** Let  $G$  be a simple graph on the vertex set  $[n] = \{1, \dots, n\}$ , that is to say,  $G$  has no loops and no multiple edges. Furthermore let  $K$  be a field and  $S = K[x_1, \dots, x_n, y_1, \dots, y_n]$  be the polynomial ring in  $2n$  variables. For  $i < j$  we set  $f_{ij} = x_i y_j - x_j y_i$ . We define the *binomial edge ideal*  $J_G \subset S$  of  $G$  as the ideal generated by the binomials  $f_{ij} = x_i y_j - x_j y_i$  such that  $i < j$  and  $\{i, j\}$  is an edge of  $G$ .

We characterize those graphs for which the quadratic generators form a Gröbner basis in a lexicographic order induced by a vertex labeling. Such

graphs are chordal and claw-free. We give a reduced squarefree Gröbner basis for general  $G$ . It follows that all binomial edge ideals are radical ideals. Their minimal primes can be characterized by particular subsets of the vertices of  $G$ .

This is joint work with J. Herzog and T. Hibi.

## References

- [1] J. Herzog, T. Hibi, F. Hreinsdottir, T. Kahle and J. Rauh *Binomial ideals and conditional independence statements*, Advances in applied mathematics, in press, available online 19. feb. 2010.

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**Speaker:** Vít Jelínek

**Affiliation:** Reykjavik University

**Title:** Pattern Avoidance in Partial Permutations

**Abstract:** One of the classical topics in combinatorics is the study of permutations that avoid a given fixed pattern. We have extended the concept of pattern avoidance in permutations to the so-called partial permutations. Informally, a partial permutation can be seen as a permutation in which some symbols have been erased and replaced by a 'wildcard' symbol that may represent any value.

We have shown that many previous results on pattern avoidance in permutations admit a non-trivial generalization to the setting of partial permutations. We have also discovered a close relationship between partial permutations and the so-called Baxter permutations. In the talk, I will give an overview of these results, and present some related conjectures.

This is joint work with Anders Claesson, Eva Jelínková and Sergey Kitaev.

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**Speaker:** Eva Jelínková

**Affiliation:** Charles University in Prague

**Title:** Forbidden Induced Subgraphs for Graph Classes in Seidel's Switching

**Abstract:** Seidel's switching is a graph operation which makes a given vertex adjacent to precisely those vertices to which it was non-adjacent before, while keeping the rest of the graph unchanged. Two graphs are called switching-equivalent if one can be made isomorphic to the other by a sequence of switches. A graph property is called hereditary if it is closed on taking induced subgraphs.

We focus on classes of graphs that are switching-equivalent to graphs with a certain hereditary property. For each such class  $C$  there exists a class  $F(C)$  of minimal forbidden induced subgraphs. Characterizations by  $F(C)$  are known for some classes, such as the finite set  $F(C)$  for graphs switching-equivalent to  $P_4$ -free graphs, and the infinite  $F(C)$  for graphs switching-equivalent to acyclic graphs.

We present a characterization of graphs switching-equivalent to  $K_{1,2}$ -free graphs by  $F(C)$  containing ten graphs, each having five vertices. We also describe the infinite  $F(C)$  for graphs switching-equivalent to forests of bounded vertex degrees.

This is joint work with Jan Kratochvíl, Department of Applied Mathematics, Charles University, Prague, Czech Republic.

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**Speaker:** Chiang Kao

**Affiliation:** National Cheng Kung University

**Title:** The quadratic assignment problem with imprecise data: The case of job assignment requiring coordination

**Abstract:** The classic quadratic assignment problem (QAP) is concerned with assigning facilities to locations. The costs involved are related to location distances and facility interactions. Different exact and heuristic algorithms have been developed for solving this type of problem. A typical QAP, which is also an extension of the traditional linear assignment problem, is job assignment requiring coordination between different jobs. Due to a lack of precise measurement, the coordination level required between different jobs and the coordination ability between assignees are not known exactly, and this paper uses fuzzy numbers to represent the imprecise values. By applying the Yager ranking technique for fuzzy numbers, the fuzzy QAP is

transformed into the conventional QAP. Thus, the existing solution methods for the QAP can be utilized to find a solution. An example of assigning the starting basketball players is used to illustrate the transformation process and the characteristics of the optimal solution.

**Keywords:** quadratic assignment problem, imprecise data, fuzzy set.

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**Speaker:** Mahdad Khatirinejad

**Affiliation:** Aalto University

**Title:** Vertex-colouring edge-weightings with two edge weights

**Abstract:** An edge-weighting vertex colouring of a (di)graph is an edge-weight assignment such that the accumulated weights at the vertices yields a proper vertex colouring. If such an assignment from a set  $S$  exists, we say the graph is  $S$ -weight colourable.

Using the Combinatorial Nullstellensatz and a classical theorem of Schur, we show that every digraph is  $S$ -weight colourable for any set  $S$  of size 2.

It is conjectured that every graph with no isolated edge is  $\{1, 2, 3\}$ -weight colourable. We explore the problem of classifying those graphs which are  $\{1, 2\}$ -weight colourable.

This is joint work with Reza Naserasr, Mike Newman, Ben Seamone, and Brett Stevens.

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**Speaker:** Torleiv Kløve

**Affiliation:** University of Bergen

**Title:** On  $B[\lambda](q)$  sets

**Abstract:** For positive integers  $\lambda$  and  $q$ , a  $B[\lambda](q)$  set  $C$  is a set of  $m$  positive integers such that all the elements  $ac$ , where  $1 \leq a \leq \lambda$  and  $c$  is in  $C$ , are incongruent modulo  $q$  (and not congruent 0). Clearly,  $1 + \lambda m \leq q$ .

It can be shown that  $B[\lambda](q)$  sets can be used to construct codes correcting single limited asymmetric errors. This is a kind of errors that can occur in flash memories.

We will present constructions of  $B[\lambda](q)$  sets and bounds on the size  $m$

of such sets. In particular, we are interested in sets where  $1 + \lambda m = q$ .

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**Speaker:** Martina Kubitzke

**Affiliation:** Reykjavik University

**Title:** NBC bases for broken circuit complexes of matroids

**Abstract:** In the first part of this talk we construct a maximal linear system of parameters for the Stanley-Reisner ring of the broken circuit complex of a regular matroid which can be described in terms of the circuits and cocircuits of the underlying matroid. In the second part we try to find monomial bases for the quotient of the Stanley-Reisner ring of the broken circuit complex with the aforementioned linear system of parameters. Two infinite classes of matroids are identified for which there exists a basis which is entirely determined by the circuits and cocircuits of the matroid. We call these bases NBC bases and show a deletion-contraction axiom for their existence. The presented results are a generalization of work by Brown, Sagan and Colbourn for broken circuit complexes of graphs.

This is joint work with Andri Egilsson.

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**Speaker:** Pekka Lampio

**Affiliation:** Aalto University

**Title:** Classification of difference matrices over cyclic groups

**Abstract:** In this computer-aided work we investigate the existence of difference matrices over cyclic groups. Up to the computational limit, we determine the maximum values of the parameters for which difference matrices exist as well as the number of inequivalent difference matrices in each case. Several new difference matrices have been found in this manner.

This is joint work with Patric Östergård.

**Speaker:** Vadim Lozin

**Affiliation:** University of Warwick

**Title:** Canonical antichains of permutations and permutation graphs

**Abstract:** We study permutations partially ordered by the pattern containment relation. This corresponds to the induced subgraph relation on the set of permutation graphs. It is well known that this partial order contains infinite antichains. We show that bipartite permutation graphs (or 321-avoiding permutations) contain a canonical antichain, i.e., an antichain  $A$  such that any subclass of bipartite permutation graphs containing finitely many graphs from  $A$  is well-partially ordered. We also show that split permutation graphs contain an antichain. However, the question whether this antichain is canonical remains an open problem.

Joint work with Colin Mayhill.

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**Speaker:** Pradipta Mitra

**Affiliation:** Reykjavik University

**Title:** Entrywise bounds for eigenvectors of random graphs and applications

**Abstract:** Let  $G$  be a graph randomly selected from  $\mathbf{G}_{n,p}$ , the space of Erdős-Rényi Random graphs with parameters  $n$  and  $p$ , where  $p \geq \frac{\log^6 n}{n}$ . Also, let  $A$  be the adjacency matrix of  $G$ , and  $v_1$  be the first eigenvector of  $A$ . We provide two short proofs of the following statement: For all  $i \in [n]$ , for some constant  $c > 0$

$$\left| v_1(i) - \frac{1}{\sqrt{n}} \right| \leq c \frac{1}{\sqrt{n}} \frac{\log n}{\log(np)} \sqrt{\frac{\log n}{np}}$$

with probability  $1 - o(1)$ . This gives nearly optimal bounds on the entrywise stability of the first eigenvector of (Erdős-Rényi) Random graphs. This question about entrywise bounds was motivated by a problem in unsupervised spectral clustering. We make some progress towards solving that problem.

**Speaker:** Włodzimierz Moczurad

**Affiliation:** Jagiellonian University

**Title:** Directed figure codes with weak equality

**Abstract:** We consider directed figures defined as labelled polyominoes with designated start and end points, equipped with catenation operation that uses a merging function to resolve possible conflicts. This is one of possible extensions generalizing words and variable-length codes to planar structures.

Within this model, we define four kinds of codes that differ in their treatment of start and end points. This is a generalization that weakens (one or both of) equality tests in the classical definition of a code. We show a strict hierarchy of those kinds and prove that testing whether a given set of figures is a code of a given kind is decidable. This is an extension of previous results, leading to a verification algorithm for codes of all four kinds.

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**Speaker:** P. Oloff de Wet

**Affiliation:** UNISA

**Title:** Simple construction of nonisomorphic graphs

**Abstract:** All graphs on  $n$  vertices can naturally be constructed by considering all binary strings of length  $\binom{n}{2}$ , where 1 indicates the presence and 0 the absence of a particular edge. We consider a similar scheme for constructing nonisomorphic graphs, called simple graph construction. This is not trivial, given the difficulty of the graph isomorphism problem (considered to be in NP but not NP-complete) and the fact that Pólya's Theorem is required to determine the number of nonisomorphic graphs. We consider a particular simple construction which creates a large number of nonisomorphic graphs, and argue that, in some sense, it may be the best possible.

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**Speaker:** Jose Plinio Santos

**Affiliation:** IMECC-Unicamp

**Title:** Combinatorial interpretation as two-line array for the Mock Theta Functions

**Abstract:** The Mock Theta Functions introduced by Ramanujan have been studied by many authors both analytically and combinatorially. The combinatorial interpretations that are known for some of them are quite different in nature. In this paper we present combinatorial interpretation as two-line array for at least 40 of them.

This is a joint work with Eduardo H. M. Brietzke and Robson da Silva.

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**Speaker:** Arseny Shur

**Affiliation:** Ural State University

**Title:** Graphs and Words: There and Back Again

**Abstract:** We exhibit a combinatorial problem which demonstrates very close interaction between graphs and words. The problem is about walks in graphs and can be easily reformulated in terms of words. We propose the solution which uses a reduction back to the walks in graphs (in a single graph now) and then an argument about words again.

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**Speaker:** Einar Steingrímsson

**Affiliation:** Reykjavik University

**Title:** Permutation patterns and the Möbius function

**Abstract:** The letters 2,6,4 in the permutation 5271643 form the pattern 132, because they appear in the same order of size as 132, that is, smallest letter first, then the largest letter and the middle one last. As another example, an occurrence of the pattern 1234 in a permutation is simply an increasing subsequence of length 4, and 5271634 avoids this pattern, that is, has no occurrence of it. Patterns in permutations have been much studied in the last few decades, and they turn out to be connected to many different combinatorial objects and various other fields, such as theoretical computer science, statistical mechanics and algebraic geometry.

The set of all permutations (of integers  $1, 2, \dots, n$  for any  $n$ ) forms a partially ordered set, where the order relation is pattern containment. For example, 132 is smaller than 5271643 in this poset, whereas 1234 is not.

An inevitable question for any poset is what the Möbius function on its intervals is. For the pattern poset, this seems to be hard for generic intervals. On the other hand, there is a computationally effective solution for the so-called separable permutations, and several results for other special cases. There are also many open problems and conjectures, and untouched aspects such as the topological properties of intervals in this poset. In short, this seems to be an interesting landscape that is only beginning to be explored. This is joint work with Bridget Tenner and with Vt Jelnek, Eva Jelneková and Alex Burstein.

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**Speaker:** Miloš Stojaković

**Affiliation:** University of Novi Sad

**Title:** How fast can Maker win?

**Abstract:** We consider positional Maker-Breaker games played on the edge set of the complete graph  $K_n$  on  $n$  vertices. Quite a few such games are known to be a Maker's win. Here we are interested in estimating the minimum number of moves needed for Maker in order to win in these games.

We will show how Maker can construct a Hamilton cycle within at most  $n + 2$  moves. This improves the classical bound of  $2n$  due to Chvátal and Erdős, and it is quite close to the trivial lower bound of  $n + 1$ . Also, we will briefly discuss games in which Maker wants to construct a spanning tree, or a perfect matching.

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**Speaker:** Casper Thomsen

**Affiliation:** Aalborg University

**Title:** On the number of zeros of multiplicity  $r$

**Abstract:** Let  $F \in \mathbf{F}[X_1, \dots, X_m] \setminus \{0\}$  be a multivariate polynomial where  $\mathbf{F}$  is any field. The multiplicity of a zero of a multivariate polynomial can

be defined; a generalization of the multiplicity of a zero of a univariate polynomial.

One can upper bound the number of zeros counted with multiplicity that  $F$  can have over  $S^m$  for some finite  $S \subseteq \mathbf{F}$  using the generalized Schwartz-Zippel bound; it was suggested by Augot et al. [1] and was recently proved by Dvir et al. [2]. The bound is in terms of the total degree of  $F$ .

We present upper and lower bounds in terms of the exponents of the leading monomial (with respect to any lexicographic ordering). The upper bound is an improvement to the generalized Schwartz-Zippel bound. Further, we allow more general point ensembles, namely  $S_1 \times \cdots \times S_m \subseteq \mathbf{F}^m$  for finite  $S_i$ .

The bounds can be hard to calculate but quite some concrete bounds can be found on <http://zeros.spag.dk>.

We present closed formulas for some cases, among others, the cases where we give sufficient conditions for when the lower bound equals the upper bound.

See [3] or <http://zeros.spag.dk> for further information.

[1] Augot, D., Stepanov, M.: "Interpolation based decoding of Reed-Muller Codes", slides from talk at Special Semester on Grbner Bases and Related Methods, RICAM, 2006, <http://www.ricam.oeaw.ac.at/specsem/srs/groeb/download/Augot.pdf>.

[2] Dvir, Z., Kopparty, S., Saraf, S., Sudan, M.: "Extensions to the Method of Multiplicities, with applications to Kakeya Sets and Mergers", arXiv:0901.2529v2, 2009, 26 pages.

[3] Geil, O., Thomsen, C.: "On the number of zeros of multiplicity  $r$ ", arXiv:0912.1436, 2009, 21 pages.

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**Speaker:** Helge Tverberg

**Affiliation:** University of Bergen

**Title:** A Lovasz-type lemma, applied to Brooks' theorem for listcolouring

**Abstract:** In 1975 Lovasz gave a new non-algebraic proof of Brooks' theorem. He first proved a lemma, saying that under certain conditions a graph  $G$  has vertices  $a, b, c$  such that  $ab$  and  $ac$  are edges, but not  $bc$ , and  $G - b - c$  is connected. One of the conditions is that  $G$  has no cutpoint. This condition can not be assumed to be satisfied by a minimal counterexample to

Brooks' theorem for listcolouring however. Since Lovasz's lemma also gives an algebraic proof of Brooks' theorem, I therefore proved a similar lemma with different conditions, giving both algebraic and non- algebraic proofs in the listcolouring case. The stimulus here was a completely different algebraic proof by Kral et.al. in 2009.

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**Speaker:** Henning Ulfarsson

**Affiliation:** Reykjavik University

**Title:** Toric permutations and pattern avoidance

**Abstract:** Usually when one studies pattern avoidance of permutations one fixes a particular pattern and counts the permutations that avoid the pattern. In this talk we will study the same counting problem when permutations are placed into equivalence classes with respect to toric equivalence. Then the sizes of the classes that entirely avoid a pattern are added up. This leads to some new and interesting counts.

We will focus on two bivincular versions of the classical pattern 213. The permutations that lie in avoiding classes are counted with Euler's  $\phi$  function and  $d(n)$  — the number-of-divisors function. This gives some interesting connections between pattern avoidance and number theory.

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**Speaker:** Kathrin Vorwerk

**Affiliation:** Royal Institute of Technology

**Title:** Connectivity of chamber graphs of finite buildings

**Abstract:** The chamber graph  $G(\Delta)$  of a thick finite building  $\Delta$ , whose edges are the pairs of adjacent chambers in  $\Delta$ , is known to be  $q$ -regular for a certain number  $q = q(\Delta)$ . We show that  $G(\Delta)$  is  $q$ -connected in the sense of graph theory. Similar results hold for chamber graphs of order complexes of geometric lattices.

This is joint work with Anders Björner.

**Speaker:** Steve Widmer

**Affiliation:** Reykjavik University

**Title:** Permutation Complexity of Words

**Abstract:** An infinite permutation  $\pi$  is a linear ordering of the natural numbers, introduced by D.G. Fon-Der-Flaass and A.E. Frid in 2007. An investigation of combinatorial complexity of the infinite permutation associated with the well-known and well studied Thue-Morse word is undertaken. A formula for the complexity is given which is found through patterns in subpermutations and the action of the Thue-Morse morphism on the subpermutations.

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**Speaker:** Patric R. J. Östergård

**Affiliation:** Aalto University/Universitt Bayreuth

**Title:** Switching Codes and Designs

**Abstract:** Switching is a local transformation that when applied to a combinatorial object gives another object with the same parameters. Switching provides a means for constructing new objects with given parameters as well as for understanding the reasons behind the multitude of isomorphism classes for certain types of objects.

Switching has been considered to some extent for virtually all kinds of combinatorial objects. Most of the results in the literature—in particular for codes and designs—can be unified by a central type of switching. For designs, switching forms a subclass of trades that can be efficiently handled computationally in an exhaustive manner.

Some specific recent results regarding switching will be mentioned. For example, it has been shown that any two Steiner triple systems of order 19 are connected to each other via a sequence of switches. As a computational challenge, this is about proving connectedness of a certain implicit graph with just over 11 billion vertices and estimated 370 billion edges.