Research Statement

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1. Introduction

In the last decade a large number of papers has been written on the subject of pattern avoidance in permutations and words, also known as the study of "restricted permutations and words" and "permutations and words with forbidden subsequences" (see [54, 86] for short surveys, and [14] for a book, on this field). My recent research is mostly dedicated to this subject, which has its roots in theoretical computer science (in the problem of sorting a permutation through a stack) and it belongs to algebraic combinatorics and combinatorics on words [2],[3],[7]–[9],[19, 21],[24, 25, 35, 36], [39]–[43], [45]–[51], [54]–[61],[63]–[65], [67], [68]; I discuss it in Section 2 alone with a brief introduction to the subject. However, I have done some research in graph theory [10, 20, 32, 44, 53, 62], in discrete analysis [1, 29, 31], in formal languages theory [38, 56], in algebra [69], and in number theory [52] not to be discussed in this statement, except for a short overview in Section 3 over representation of graphs coming from study of growth of free spectrum algebraic problems.

Some of my most valuable achievements, mostly mentioned in Section 2 in slightly more details, can be summarized as follows:

(1) As a corollary to a much more general theorem, in [46], I found an explicit generating function for the distribution of peaks in permutations, a classical object studied for long time (the distribution was known before in terms of continued fractions only).
(2) In [40], I derived a generating function for the entire distribution on permutations of the maximum number of non-overlapping occurrences of an arbitrary segmented pattern in terms of the exponential generating function for the number of permutations avoiding the pattern.
(3) I introduced the notion of "partially ordered patterns" that allows us to provide a uniform notation for several combinatorial structures studied in the literature (see [40, 46, 47]).
(4) In [38], I proved that the Arshon sequence (introduced by Arshon in 1937) cannot be generated by a morphism.
(5) In [41], I obtained a new simple identity involving the Bell numbers and the Stirling numbers of the second kind.

Moreover, in collaboration with other people, the following results were obtained.

(1) In [7], (2+2)-free posets (a well-studied object due to its connection to the interval orders) were enumerated. Further, in [66], these posets were enumerated subject to 4 statistics including the number of minimal elements in posets.
(2) In [69], the word problem for Perkins semigroup was solved in terms of alternation words digraphs. Perkins semigroup has played central role in semigroup theory since 1960, particularly as a source of examples and counterexamples.
(3) In [7], a conjecture of Pudwell on certain class of permutations was settled.
(4) In [19], a classification of bijections between 321- and 132-avoiding permutations was obtained. The first such bijection was introduced by Knuth in 1969, and this was the beginning of permutation patterns theory.
(5) The most general framework in modern permutation pattern theory is suggested in [65].
(6) In [21], one finds a class of permutations defined by avoidance of generalized patterns which is equinumerous to the class of 2-stack sortable permutations (TSS), rooted nonseparable planar maps, and $\beta(1,0)$-trees. TSS is one of the classical objects in

permutation patterns theory, but the class of permutations we discovered in [21] has much nicer structure compare to TSS.

2. Patterns in permutations, words and matrices

We write permutations as words $\pi = a_1 \cdots a_n$, whose letters are distinct and usually consist of the integers $1, \dots, n$. An occurrence of a *pattern* τ in a permutation π is a subsequence in π (of the same length as τ) whose letters are in the same relative order as those in τ.

We denote by $\mathcal{S}_n(\tau)$ the set of all permutations in $\mathcal{S}_n$ which *avoid* τ, that is, have no occurrences of τ. If $R = \{\tau_1, \dots, \tau_m\}$, we let $\mathcal{S}_n(R) = \bigcap_{1 \le i \le m} \mathcal{S}_n(\tau_i)$. Fundamental questions are to determine $|\mathcal{S}_n(R)|$ viewed as a function of n, and if $|\mathcal{S}_n(R)| = |\mathcal{S}_n(R')|$ to find an explicit bijection between $\mathcal{S}_n(R)$ and $\mathcal{S}_n(R')$. It is also interesting to find relations between $\mathcal{S}_n(R)$ and other combinatorial structures. By determining $|\mathcal{S}_n(R)|$ we mean finding an explicit formula, or *ordinary* or *exponential generating functions* (*GF* and *EGF* respectively). A more general question is to find the *distribution* of a given pattern τ, that is, to find out how many permutations of length n have exactly k occurrences of τ for any n and k.

In [4] Babson and Steingrímsson introduced *generalized permutation patterns* that allow the requirement that two adjacent letters in a pattern must be adjacent in the permutation. If we write, say 2-31, then we mean that if this pattern occurs in a permutation π, then the letters in π that correspond to 3 and 1 are adjacent. For example, the permutation $\pi = 516423$ has only one occurrence of the pattern 2-31, namely the subword 564, whereas the pattern 2-3-1 occurs, in addition, in the subwords 562 and 563.

The motivation for introducing these patterns in [4] was the study of *Mahonian statistics*. Many interesting results on generalized patterns appear in the literature (see, e.g., [5, 11, 12, 13, 17, 22, 26, 27, 30], [75]–[81]). In particular, [17] provides relations of generalized patterns to several well studied combinatorial structures, such as set partitions, *Dyck paths*, *Motzkin paths* and involutions.

2.1. **My research in permutation patterns.**

It is a classical result, first established by Knuth [70] in 1969, that the number of 3-2-1-avoiding permutations is equal to that of 2-3-1-avoiding permutations and both are given by *Catalan numbers*. In the literature one can find many subsequent bijective proofs confirming this fact. In [19] we classify the existing bijections showing which of them are equivalent modulo trivial bijections and how many permutation statistics each of the bijections preserves. Moreover, in [19] we introduce a recursive description of the algorithmic bijection given by Richards [82] in 1988 (combined with a bijection by Knuth from 1969). This bijection respects 11 statistics (the largest number of statistics any of the bijections respects).

A formula for the number of permutations of length n that can be sorted by passing them twice through a *stack* (where the letters on the stack must be in increasing order) was conjectured by West, and later proved by Zeilberger. Goulden and West found a bijection from such permutations to *rooted nonseparable planar maps*, and later, Jacquard and Schaeffer presented a bijection from these planar maps to certain labeled plane tress, called $\beta(1,0)$*-trees*. Using generating trees, Dulucq, Gire and West showed that nonseparable planar maps are equinumerous with permutations avoiding 2-4-1-3 and the *barred pattern* 4-1-$\bar{3}$-5-2; they called these permutations *nonseparable*.

In [21] we give a new bijection, preserving 7 statistics, between $\beta(1,0)$-trees and permutations avoiding 3-1-4-2 and 2-41-3 which can be seen to be exactly the reverse of nonseparable permutations. As a corollary to this bijection, we give a new bijection between nonseparable permutations and rooted nonseparable planar maps. In connection with this we give a non-trivial involution on the $\beta(1,0)$-trees, which specializes to an involution on unlabeled rooted plane trees, where it yields interesting results.

2.2. **Miscellaneous results on pattern avoidance.** In [39] we consider 3-patterns with no dashes (*consecutive* 3-patterns), that is, patterns corresponding to contiguous subwords anywhere in a permutation. Such patterns have also been considered by other authors, e.g., in [11, 13, 26, 27, 79, 80]. As [83] deals with multi-avoidance of "classical" patterns (introduced at the very beginning of Section 2), in [39] we give either an explicit formula or a recursive formula for many of cases of simultaneous avoidance of more than one consecutive 3-patterns. The remaining cases were solved in [49, 50]. In [25] we suggest an analytic approach using the spectral theory of integral operators on $L^2([0,1]^m)$ (where $m+1$ is the length of prohibited patterns) to study asymptotics for consecutive patterns. Our methods give detailed asymptotic expansions and allow for explicit computation of leading terms in many cases. We obtain in a different way, and improve, some of the results from [27]. In general, there are several approaches to study occurrences of consecutive patterns in permutations. In [2] we propose yet another approach which is based on considering the *graph of patterns overlaps*, which is a certain subgraph of the *de Bruijn graph*.

In [41, 49, 50] we consider avoidance of generalized 3-patterns with additional restrictions. The restrictions consist of demanding that permutations in question must begin and/or end with certain patterns. One motivation for considering such additional restrictions is their connection to some classes of trees (see [85, p. 24] for encoding permutations by certain trees). For instance, the 132-avoiding n-permutations that begin with the pattern 12 are in one-to-one correspondence with the *increasing rooted trimmed trees* with $n+1$ nodes. (In an increasing rooted tree, the nodes are numbered and the numbers increase as we move away from the root; a trimmed tree is a tree where no node has a single leaf as a child.) Also, in [41] we get relations to partitions of special type, and obtain a new simple identity involving the *Bell numbers* and the *Stirling numbers* of the second kind:

$$\sum_{i=0}^{n-1}\binom{n}{i}B_i=\sum_{i=1}^{n} i\cdot S(n,i).$$

2.3. **An extension of generalized patterns.** We define the following class of permutations:

$$\mathcal{R}_n=\{\pi_1\ldots\pi_n\in\mathcal{S}_n \mid \text{if } \pi_i\pi_j\pi_k \text{ forms 2-3-1 then } j\neq i+1 \text{ or } \pi_i\neq\pi_k+1\}.$$

Essentially, it is a class of permutations that avoid a particular pattern of length three. This type of pattern is new in the sense that it does not admit an expression in terms of generalized patterns. An attractive property of these patterns is that, like classical patterns, they are closed under the action of D_8, the symmetry group of the square.

In [7] (see also [18] for a relevant paper) we present bijections between four classes of combinatorial objects. Two of them, the class of *unlabeled (2+2)-free posets* and a certain class of *chord diagrams* (or *involutions*), already appear in the literature. The third one is $\mathcal{R}_n$. The fourth class is formed by certain integer sequences, called *ascent sequences*. Our bijections preserve numerous statistics. Moreover, in [7] we determine the generating function of these classes of objects, thus recovering a series obtained by Zagier for chord diagrams. The fact that this series counts (2+2)-free posets is new. In any case, in [66] we extend the enumerative result by finding the generating function for (2+2)-free posets when four statistics are taken into account, one of which is the number of minimal elements in a poset. Also, in [7] we characterize ascent sequences that correspond to permutations avoiding the barred pattern 3-$\bar{1}$-5-2-$\bar{4}$, and enumerate those permutations, thus settling a conjecture of Lara Pudwell. Finally, in [66] we use ascent sequences to give an alternative proof of the known fact ([85, 84]) that the number of posets avoiding simultaneously (2+2) and (3+1) is given by Catalan numbers.

2.4. **Partially ordered patterns in permutations and words.** In [40, 46] we introduce a further generalization of generalized patterns (GPs), namely *partially ordered generalized patterns* (*POGPs*). As in [46, 47, 59], in what follows, POGP will be abbreviated by *POP* staying for a *partially ordered pattern.* A POP is a GP some of whose letters are incomparable. For instance, if we write $\tau = 1\text{-}1'2'$ then we mean that in an occurrence of τ in a permutation π the letter corresponding to the 1 in τ can be either larger or smaller than the letters corresponding to $1'2'$. Thus, the permutation 31254 has three occurrences of τ, namely 3-12, 3-25, and 1-25. As the matter of fact, some POPs appeared in the literature before they were actually introduced in [40, 46] (see, e.g., [6, 37]). Thus the notion of a POP allows us to collect under one roof (to provide a uniform notation for) several combinatorial structures such as *peaks*, *valleys*, *modified maxima* and *modified minima* in permutations, *Horse permutations*, *p-descents* and others.

Let $\mathcal{S}_\infty$ denote the disjoint union of the $\mathcal{S}_n$ for all $n \in \mathbb{N}$. The POPs (which include the GPs, as well as the classical patterns), can be considered as functions from $\mathcal{S}_\infty$ to $\mathbb{N}$ that count the number of occurrences of a fixed pattern in a permutation in $\mathcal{S}_\infty$. This allows us to write a POP (as a function) as a linear combination of GPs. For example,

$$1'\text{-}2\text{-}1'' \;=\; (1\text{-}3\text{-}2) \;+\; (2\text{-}3\text{-}1),$$

from which, in particular, we see that to avoid $1'\text{-}2\text{-}1''$ is the same as to avoid simultaneously the patterns 1-3-2 and 2-3-1. The example above demonstrate how closely the POPs are related to the GPs. In fact, dealing with POPs we deal with sets of GPs.

In any case, the motivation for introducing POPs in [40, 46] is that they allow us to find the EGF for the *entire distribution* of the maximum number of non-overlapping occurrences of a pattern τ with no dashes, if we only know the EGF for the number of permutations that avoid τ (we do know the EGF for many GPs with no dashes due to [27]). To be more precise, the following theorem holds.

Theorem 2.1. *Let τ be a consecutive pattern. Let $A(x)$ be the EGF for the number of permutations that avoid τ. Then*

$$\sum_{\pi} y^{\tau\text{-}nlap(\pi)} \frac{x^{|\pi|}}{|\pi|!} = \frac{A(x)}{1 - y((x-1)A(x)+1)}$$

where τ-$nlap(\pi)$ is the maximum number of non-overlapping occurrences of τ in π.

An alternative, more complicated though, proof of Theorem 2.1, as well as that of Theorem 2.4 below, appears in [79, 80]. Theorem 2.1 is a starting point in [79, 80] where it is proved by exploiting the relationship between the elementary and homogeneous symmetric functions. Also, the class of permutation patterns for which Theorem 2.1 holds is enlarged in [79, 80]. Moreover, in [46, 79, 80] a q-analogue for Theorem 2.1 is found.

Also, in [40] we give alternative proofs, using inclusion-exclusion, of some of the results in [27]. Our proofs result in explicit formulas for the coefficients of the EGF whereas in [27] the authors obtained differential equations for these EGF. One more result in [40] is worth mentioning (it is cited in [71] by Knuth, who actually introduced the concept of a permutation pattern in [70]):

Theorem 2.2. *Let $\tau = \sigma$-k, where σ is an arbitrary consecutive pattern on the elements $1, \ldots, k-1$. So, the last element of τ is greater than any other element. Let $A(x)$ (resp. $B(x)$) be the EGF for the number of permutations that avoid σ (resp. τ). Then $B(x) = e^{F(x,A(y))}$, where $F(x, A(y)) = \int_0^x A(y)\ dy$.*

Theorem 2.2 is extended in [34] to dealing with the pattern $\tau = k\text{-}\sigma\text{-}k$. A corollary to Theorem 2.2 is considered in the following example.

Example 2.3. *Suppose* $\tau = 12\text{-}3$. *Here* $\sigma = 12$, *whence* $A(x) = e^x$, *since there is only one permutation that avoids* σ. *So*

$$B(x) = e^{F(x,e^y)} = e^{e^x - 1},$$

which agrees with a result in [17] *that the number of* n*-permutations that avoid* τ *is the* n*-th Bell number whose EGF is* $B(x)$.

As a corollary to a much more general theorem on certain POPs, in [46] we find a bivariate generating function for the distribution of peaks in permutations (x is responsible for length of permutations and y for the number of occurrences of peaks):

$$1 - \frac{1}{y} + \frac{1}{y}\sqrt{y-1} \cdot \tan\left(x\sqrt{y-1} + \arctan\left(\frac{1}{\sqrt{y-1}}\right)\right).$$

Even though the "peaks in permutations" is a classical combinatorial object, the generating function for the peaks' distribution was only known in terms of a continued fraction while we provided an exact formula for it.

Further study of POPs in permutations is conducted in [45], where we not only prove a result from [39] in a much simpler way, but also establish a connection between POPs with no dashes and walks on lattice points starting from the origin and remaining in the positive quadrant. Also, in [45] we find the EGF for the number of permutations avoiding consecutive POPs of length 4 in few cases. One example of the results in this direction, which actually appears in [40], is the EGF for the number of permutations that avoid the pattern $122'1'$:

$$\frac{1}{2} + \frac{1}{4}\tan x(1 + e^{2x} + 2e^x \sin x) + \frac{1}{2}e^x \cos x.$$

In [62] we study a special type of POPs, called V- and Λ-patterns, which generalize valleys and peaks, as well as *increasing* and *decreasing runs*, in permutations. A complete classification of permutations (multi)-avoiding V- and Λ-patterns of length 4 is given. We also establish a connection between restricted permutations and matchings in the *coronas of complete graphs*.

In [9] we study the encoding of several combinatorial objects by POPs. That is, given a class of objects, say, a class T_n of certain graphs on n nodes, we find a set of POPs to avoid in n-permutations which gives the same cardinality as the cardinality of T_n. This idea was used in [59] to enumerate occurrences of consecutive patterns in *compositions* (a composition, after removing all "+" signs, can be viewed as a word over the alphabet of natural numbers; so we may study occurrences of patterns in such words).

In [52] we extend the concept of POPs in permutations to that in words. We give analogies, extend and generalize several known results, and get some new results. In particular, we give the GF for the entire distribution of the maximum number of non-overlapping occurrences of a pattern τ with no dashes (and possibly with repeated letters) in k-ary words, provided we know the GF for the number of k-ary words that avoid τ (we do know the GF for many of such patterns due to [11]–[13]). We state the main result in [52] as the following theorem. One can compare it with Theorem 2.1.

Theorem 2.4. *Let* τ *be a consecutive pattern and let* $A_\tau(x;k) = \sum_{n\geq 0} a_\tau(n;k)x^n$ *be the generating function for the numbers* $a_\tau(n;k)$ *of words in* $[k]^n$ *avoiding the pattern* τ. *Then for all* $k \geq 1$,

$$\sum_{n\geq 0}\sum_{\sigma\in[k]^n} y^{\tau\text{-}nlap(\sigma)}x^n = \frac{A_\tau(x;k)}{1 - y((kx-1)A_\tau(x;k)+1)}$$

where τ-nlap(σ) is the maximum number of non-overlapping occurrences of τ in σ.

A q-analogue to Theorem 2.4 is found in [36]. It is based on considering patterns in compositions.

2.5. **Other notions of a "pattern".** In [43, 58, 68] we generalize the concept of pattern occurrence in permutations and words first to that in matrices and then to pattern occurrences in n-dimensional objects, which are basically sets of $(n+1)$-tuples. For certain patterns or sets of patterns related to our objects, we get an unexpected connection to Ramsey Theory, which makes our research interesting from a graph theoretic point of view. For example, occurrences of particular matrix patterns in the adjacency matrix of a graph pose certain restrictions on the set of edges of the graph and on its cycles. The *bipartite Ramsey numbers* and the concept of "*vanishing borders*" arise in our study. In some cases, we employ direct combinatorial considerations to obtain either explicit closed formulas or generating functions; in other cases, we use the transfer matrix method to derive an algorithm which gives a closed formula when a certain parameter is fixed.

One more way to proceed with patterns is to pay attention to parity of elements. In [63] we refine the well-known permutation statistic "descent" by fixing parity of (exactly) one of the descent's numbers. We provide explicit formulas for the distribution of these (four) new statistics. We use certain differential operators to obtain the formulas. Moreover, we discuss connections of our new statistics to the *Genocchi numbers*, the study of which goes back to Euler. We also provide bijective proofs of some of our results from this paper. In [64] we have generalized the results of [63] to classify descents according to equivalence mod k for $k \geq 3$. As a result of our study, we obtain, for example, the following remarkable identities for all $n \geq 0$, $k \geq 2$, and $0 \leq s \leq n$:

$$\left[\sum_{r=0}^{s}(-1)^{s-r}\binom{(k-1)(n+1)+r}{r}\binom{kn+k}{s-r}\prod_{i=0}^{n-1}(r+1+(k-1)(i+1))\right] =$$
$$\left[\sum_{r=0}^{n-s}(-1)^{n-s-r}\binom{(k-1)(n+1)+r}{r}\binom{kn+k}{n-s-r}\prod_{i=0}^{n-1}(r+(k-1)(i+1))\right].$$

The papers [63] and [64] can be viewed as a first step in a more general program which is to study pattern avoiding conditions on permutations where generalized parity considerations are taken into account. The work in this direction is continued in [33, 65, 73, 74]. However, the most general framework in modern permutation pattern theory is suggested in [65] (not to be discussed in this statement).

2.6. **More on my research in patterns.** Most attention, in the papers on classical or generalized patterns is given to finding exact formulas and/or generating functions for the number of words or permutations avoiding a given pattern τ, or having exactly k occurrences of τ. In [42, 56, 57] we suggest another problem, which is to count the occurrences of certain patterns in certain words. These words were chosen to be the set of all finite approximations of certain sequences. In [57] we start a general study of counting the number of occurrences of patterns in words generated by morphisms. In [56] we treat the sequence obtained from the *Peano curve*. The Peano curve was studied by Peano in 1890 as an example of a continuous space filling curve. Finally, in [42] we study the *sigma-sequence*, which was used by Evdokimov [28] to construct chains of maximal length in the n-dimensional unit cube. The sigma-sequence is interesting, e.g., in connection with the well-known *Dragon curve*, discovered by physicist John E. Heighway.

When defining or characterizing sets of objects in discrete mathematics, "languages of prohibitions" are often used to define a class of objects by listing the prohibited subobjects, i.e., subobjects that are not allowed to be contained in the objects of the class. The notion of a subobject is defined in different ways depending on the objects under consideration: a subword for fragmentarily restricted languages, a subgraph for families of graphs, a subshape for two-dimensional shapes and so on.

We collect all prohibited objects into a set that we call a *set of prohibitions*. The idea of *unavoidable set* is as follows: if there exists a restriction on the size of an object, in other words, if large enough objects must contain prohibited subobjects, then the set of prohibitions is unavoidable. In [8] we extend the concept of sets of prohibited words to that of prohibited *word patterns*. A word pattern contains each of the letters $1, \ldots, k$ at least once for some k, and no other letters. For instance, the word 2613235 contains an occurrence of the pattern 1323, but its factor 2613 is not a pattern. In [8] we not only study numerical characteristics of sets of prohibited word patterns, but also give a link to intensively studied *universal cycles* introduced in [16]. We prove that there is a sequence (universal cycle) containing each word pattern as its factor exactly once for any fixed length and over any fixed alphabet. As a corollary to our results, we obtain a combinatorial proof of the following (new) identity

$$\sum_{i=1}^{m} i!S(n,i) = \sum_{i|n} \sum_{j=0}^{\min(i,m)-1} (-1)^j \binom{\min(i,m)-1}{j} \sum_{d|i} \mu(d)(\min(i,m)-j)^{\frac{i}{d}}$$

where $S(n,i)$ is a Stirling number of the second kind and $\mu(n)$ is the Möbius function defined as

$$\mu(n) = \begin{cases} 0, & \text{if } n \text{ has one or more repeated prime factors,} \\ 1, & \text{if } n = 1, \\ (-1)^k, & \text{if } n \text{ is a product of } k \text{ distinct primes.} \end{cases}$$

3. Representation of graphs

The notion of representable graphs was introduced in [69] to obtain asymptotic bounds on the *free spectrum* of the widely-studied *Perkins semigroup* which has played central role in semigroup theory since 1960, particularly as a source of examples and counterexamples. This notion is interesting by itself combinatorially and we study it from this point of view in [32, 61]. Also, representable graphs are studied in [72]. In the rest of the section, we provide a brief overview over selected results on representable graphs together with some basic definitions.

A graph $G = (V, E)$ is *representable* if there exists a word W over the alphabet V such that letters x and y alternate in W if and only if $(x, y) \in E$ for each $x \neq y$. If W is *k-uniform* (each letter of W occurs exactly k times in it) then G is called *k-representable*. It was shown in [61] that a graph is representable if and only if it is k-representable for some k. We call a graph *permutationally representable* if it can be represented by a word of the form $P_1 P_2 \ldots P_k$ where all P_i are permutations. In particular, all permutationally representable graphs are k-representable. It was shown in [69] that a graph is permutationally representable if and only if it is a comparability graph. In particular, all bipartite graphs are permutationally representable.

For a vertex $x \in V(G)$ denote by $N(x)$ the set of all its neighbors. It was shown in [61] that if G is representable, then for every $x \in V(G)$ the graph induced by $N(x)$ is permutationally representable. The converse to the last statement is shown to be false in [32].

The *representation number* of a graph G is the minimum k such that G is k-representable. According to [61], the *triangular prism* have representation number 3, while the *outerplanar* *graphs* are 2-representable. In fact, it was shown in [32] that a graph is 2-representable if and only if it is a *circle graph*. Moreover, the following theorem is proved in [61]:

Theorem 3.1. *For every graph G there exists a 3-representable graph H that contains G as a minor. In particular, the 3-subdivision of every graph G is 3-representable.*

In [32], we give an example of k-representable but not $(k-1)$-representable graph thus showing that there does not exist a constant c such that each representable graph is a-representable for $a \leq c$. Moreover, we show, in [32], that each representable graph on n-nodes is n-representable.

The main result in [32] is a characterization of representable graphs in terms of orientations. Namely, we show that a graph is representable if and only if it admits an orientation into a *semi-transitive digraph* (see [32] for details).

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