FINALLY, WE CAN COUNT 2+2!

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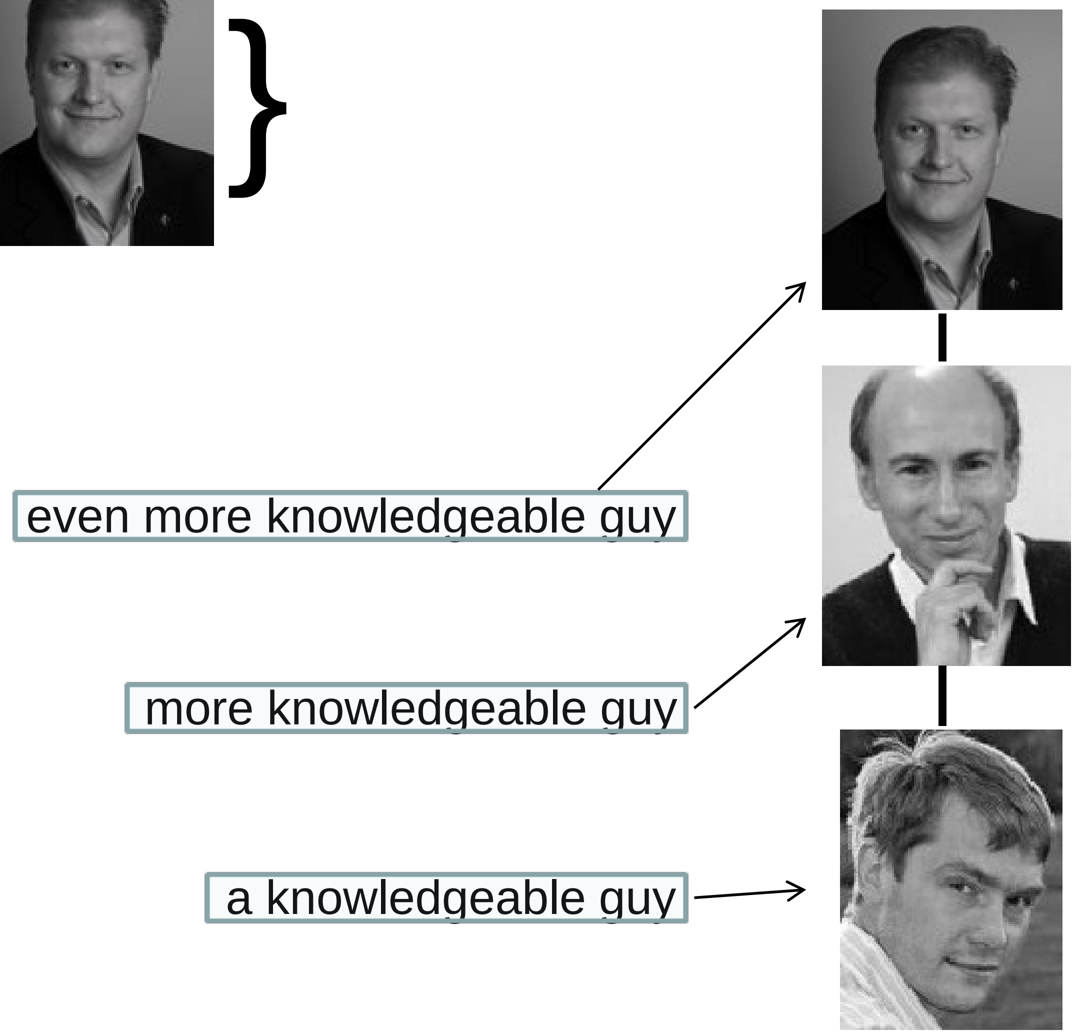
Enumerative combinatorics

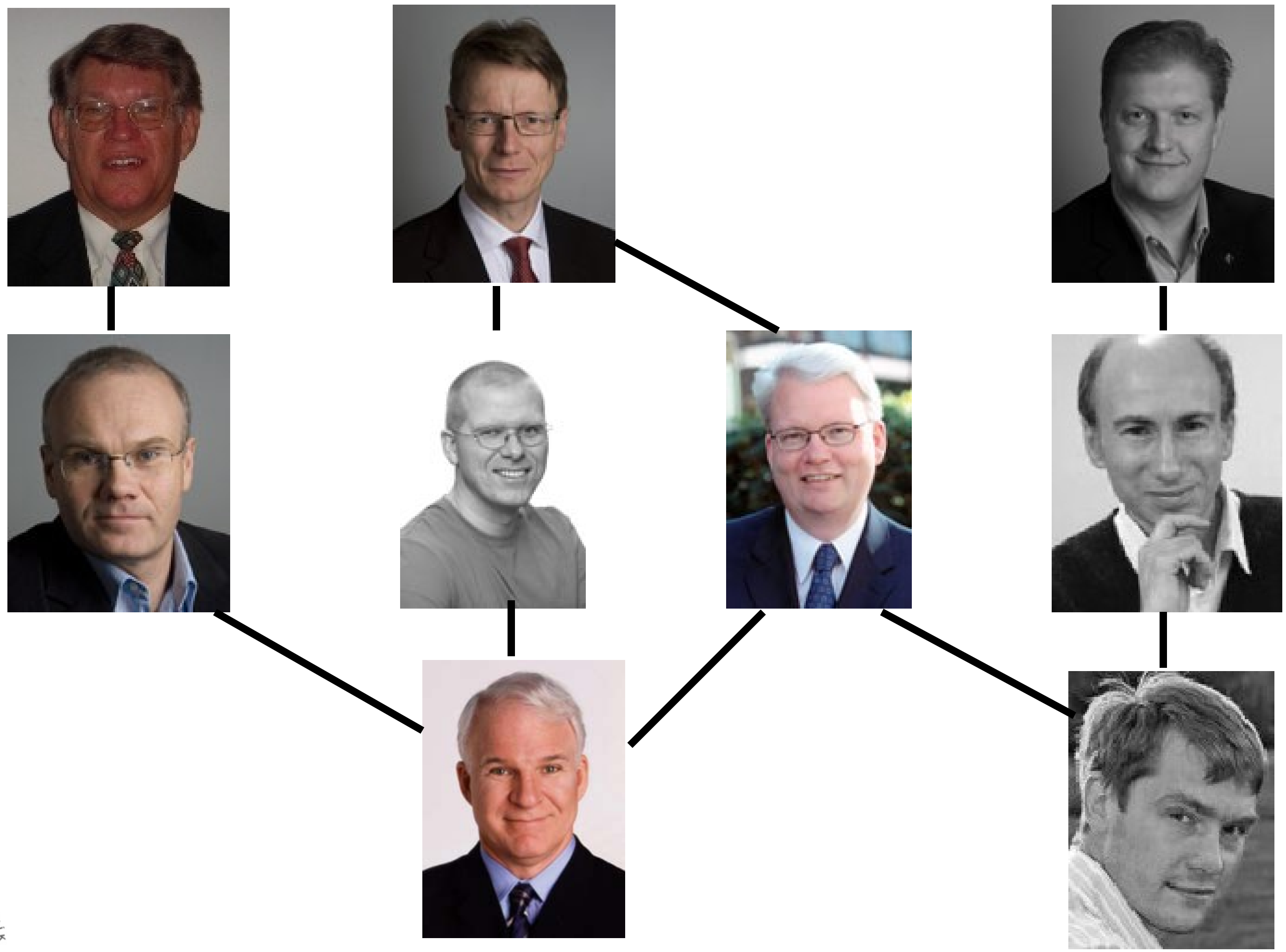
Sample questions:

- How many people are in this room?
- In how many different ways can we choose two people to leave the room?

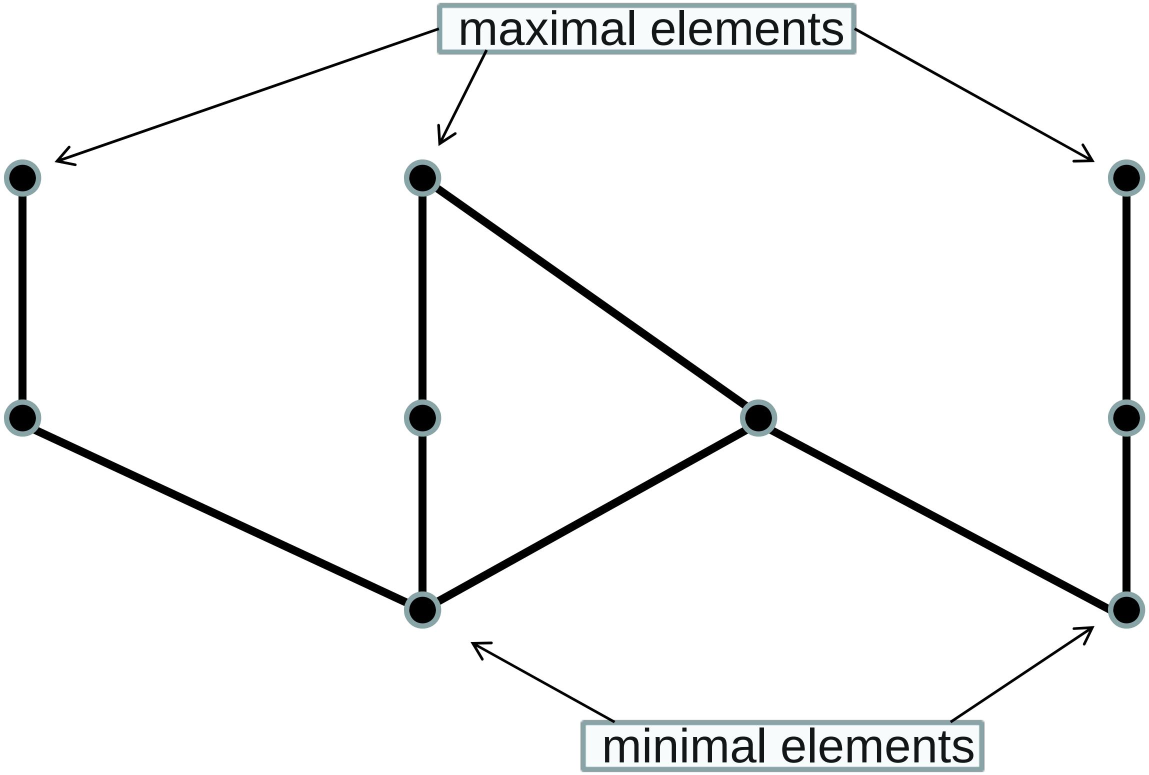
Totally ordered sets



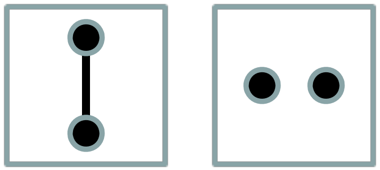
Partially ordered sets (posets)



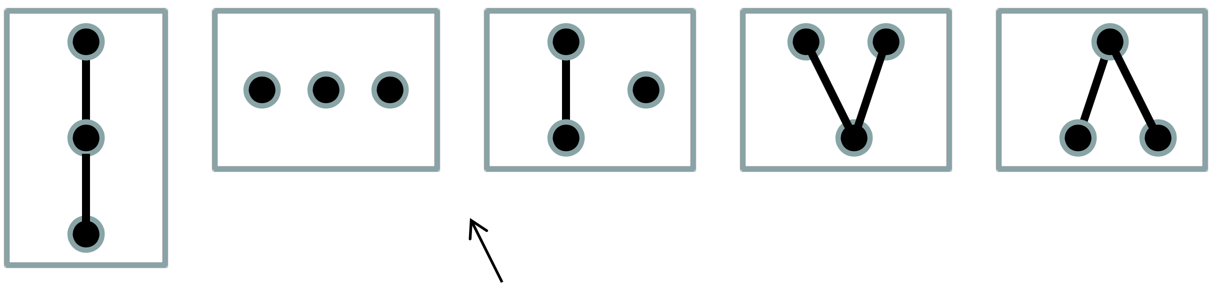
Partially ordered sets (posets)



Counting (unlabeled) posets



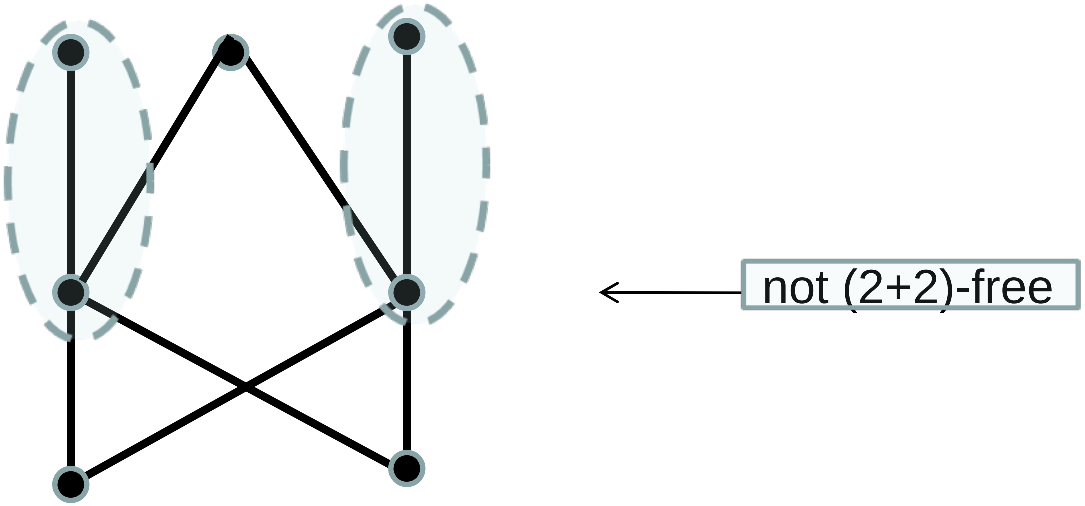
2 posets on 2 elements

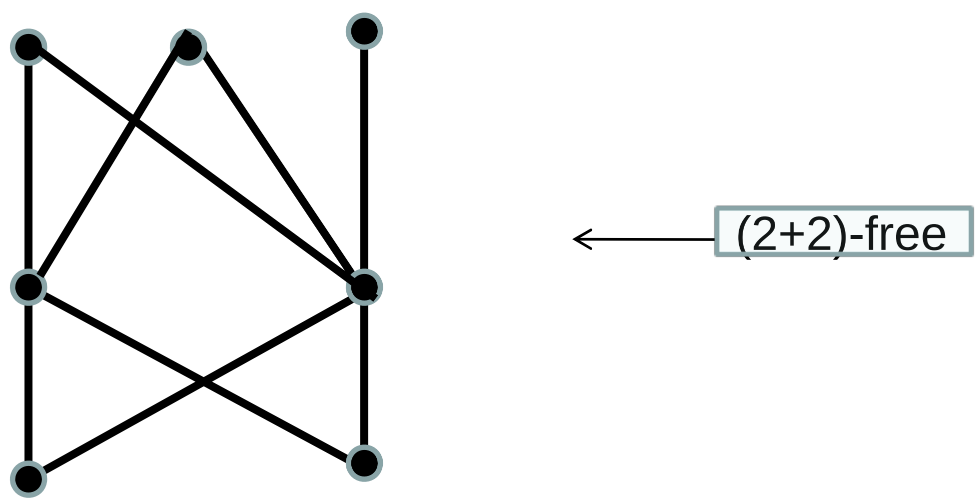


5 posets on 3 elements

Unknown: number of posets on 15 or more elements.

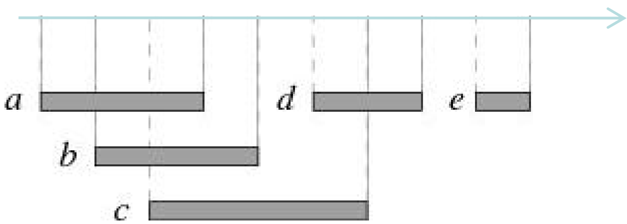
(2+2)-free posets

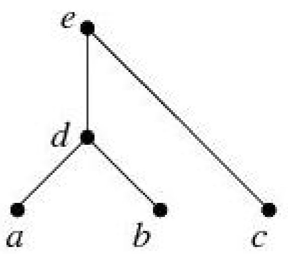




(2+2)-free posets

(2+2)-free posets arise as interval orders (Fishburn):





P. C. Fishburn, Intransitive indifference with unequal indifference intervals, *J. Math. Psych.* ***7 (1970) 144–149.***

Actually, (2+2)-free posets are connected to at least 3 other objects studied in the literature and they also appear in some applications.

How many (2+2)-free posets are there?

Bousquet-Melou, Claesson, Dukes, and Kitaev:

The generating function of unlabeled $(\mathbf{2}+\mathbf{2})$-free posets is

$$P(t) = \sum_{n\geq 0} \prod_{i=1}^{n} \left(1-(1-t)^i\right).$$

Kitaev and Remmel:

$$\begin{aligned} G \;=\; & \frac{1}{1-tz} + \frac{ut}{v-1-tv(1-u)} \sum_{r\geq 1} (tz)^r ((v-1)uv + \\ & \frac{((v-1)z-v)(1-u)t}{(1-zt(1-u))(1-t(1-u))} \left(1+\sum_{k\geq 1} \frac{(u(1-t))^k \prod_{i=0}^{k-1}(u+(1-u)(1-t)^i)}{\prod_{i=1}^{k}(u+(1-u)(1-t)^i(1-zt))(u+(1-u)(1-t)^{i+1})}\right) + \\ & \frac{uv^3t(1-uv)}{(1-zt(1-uv))(1-t(1-uv))} \left(1+\sum_{k\geq 1} \frac{(uv(1-t))^k \prod_{i=0}^{k-1}(uv+(1-uv)(1-t)^i)}{\prod_{i=1}^{k}(uv+(1-uv)(1-t)^i(1-zt))(uv+(1-uv)(1-t)^{i+1})}\right)). \end{aligned}$$

The end.