Invited Paper: Models for Wireless Algorithms

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***Abstract*—To develop algorithms and protocol for wireless mesh networks, one needs good models, especially capturing the defining aspect of the wireless medium: interference. The model should be simple and general, yet realistic. We survey here the various modeling aspects, focusing particularly on recent work involving physical models.**

I. Introduction

The distinguishing aspect of wireless communication is the existence of *interference*. This involves both the *signal propagation*, how the transmission travels, so to speak, and the extent to which other simultaneous communication disturbs the reception.

Algorithms (or protocols) need a firm ground on which to base their decisions. In usual parlance, this is called a *model*: what actions are possible and what will be their impact. The algorithms can be evaluated either by simulations or by theoretical analysis; in both cases, it is crucial to have a clear model for determining which transmissions can be properly received.

Good models have certain desirable traits. They should be *general* enough to work in a wide variety of settings and mostly free from technology-specific attributes. They should be *simple* enough to be usable by algorithms, and possible to reason about, i.e., be *analyzable*. Finally, we expect them to have high *fidelity*, being a close (enough) match to reality. There can easily be tension between these properties, suggesting possible tradeoffs. In many settings, the elegance and greater abstraction of simpler models, along with the potential greater ease of implementation, may be deemed worth the resulting loss of fidelity. Studying the simpler models first is also a way to uncover the intrinsic properties that guide the way for the more detailed models.

The right model necessarily depends on the problems or questions being asked. We will focus here on problems involving, fully or partially, the MAC (Media Access Control) layer, which handles scheduling of communication to avoid or respond to interference, which is generally abstracted away by higher layers. The modeling can easily involve the PHY layer, which defines the lower-level abilities, and the problems can also depend on cross-layer issues involving the network layer, so the modeling task is not solely restricted to the MAC layer. The fundamental problems involved have to do with the capacity of wireless network: how much communication can we have going on simultaneously, and what throughput can be handled, under different communication patterns.

Our focus is particularly on the big picture, behavior-at-large, or the scalability asymptotics. We also prefer to avoid assumptions about input distributions, seeking performance guarantees, or "every-case" properties.

We survey in this paper models that have been proposed or studied in recent years. The focus here is on *physical models* (or fading-channel models), where interference is not a binary, pairwise relationship that can be captured by unweighted graphs. We especially omit models that primarily focus on capturing connectivity, as opposed to interference, most of which are graph-based. For these, we refer to a comprehensive survey of Schmid and Wattenhofer [19] from 2006.

In the next section, we introduce the established models of the field, but point out the challenges raised by actual wireless reception conditions. Recent models and ideas from the computer science theory literature for tackling these are outlined in Sec. III, while the more established approaches of adding stochastic components are given in Sec. IV. Distributed algorithms require additional consideration, treated in Sec. V. Various other aspects are covered briefly in Sec. VI.

We describe numerous Models and Special Cases, identify Options to those models and point out essential Properties.

II. Standard Models

There are two major families of models of wireless interference. One considers only the worst interfering transmission, which gives rise to a binary relationship that can be captured by (undirected, unweighted) graphs. The other takes all the interfering transmissions into account, summing up their strengths as received. The standard assumption in both cases is that the wireless nodes are located in the Euclidean plane.

A. Range models

The protocol model proposed in [7] is the most heavily used model in analytic works.

Model 1 (Protocol Model (PM))**.** *Each node v has a* transmission radius r_v *and* interference radius $\lambda \cdot r_v$*, for some constant* $\lambda \geq 1$*. The transmission of v to w is successful if w is within the transmission radius from v and outside the interference radius from every other simultaneously transmitting node u, i.e.,* $d(v, w) \leq r_v$ *and* $d(v, u) > \lambda \cdot \max(r_v, r_u)$.

The transmission radii are generally assumed to be proportional to the reception power (transmission power to the appropriate power), namely $r_v = c \cdot d(v, v')^{\alpha}$, where c is a constant, v' is the intended receiver of v's transmission, and α is the "pathloss constant" explained later.

The model is frequently used with uniform power (UNIFORM), in which case the transmission radii are uniform. This special case is also known as *Unit-Disk-Graphs with Distance Interference (UDI)* in [19].

Special Case 1 (Uniform-Radius Protocol Model (URP))**.** *Protocol model where the transmission radius of each node is the same.*

B. Physical models

The physical models eschew the binary relations of range models, viewing signals as decreasing real-valued quantities that should be compared in terms of relative strengths. There are three constants that play a role here: α is the "pathloss exponent", usually considered to be in the range $(2, 6)$; β, a parameter of the hardware and coding functions; and N, the ambient noise term.

Model 2 (Geometric SINR model (GEOSINR))**.** *The model is based on three axioms: (i)* The pathloss (or, the relative decrease) of a signal is a polynomial of the distance traveled. *I.e., if v transmit at power P_v, then it is heard at u with power $P_{uv} = P_v/d(u, v)^\alpha$. (ii)* Interference adds up. *The total interference at node u from a set S of transmitters is $\sum_{v\in S} P_{vu}$. (iii)* The success of a reception is a threshold function of the signal-to-interference ratio. *Namely, transmission from v to w succeeds if $P_{vw} \geq \beta \cdot (N + \sum_{u\in S} P_{uw})$, where S is the set of other transmitters.*

The issue of power control is crucial in these models. It is not only a question of whether power control is possible, but also whether it is under the control of the algorithm (on a per transmission basis), and then based on what information.

Option 1 (Power Control)**.** UNIFORM*: All nodes transmit with the same power.* OBLIVIOUS*: The power level depends on the distance d between sender and receiver. Specifically, for some constant $\tau \in [0, 1]$, the power is proportional to $d^{\tau\alpha}$.* GENERAL*: The power setting is under control of the algorithm.* FIXED*: Each node has a fixed power, that is not necessarily related to the transmission distance.*

C. Issues: Wireless reality

Real environments behave quite differently from the nice geometric representations in the standard models. Signals are reflected or diffracted by ground, ceilings, walls and objects, which causes it to travel along multiple path, which results in kaleidoscopic patterns of constructive and destructive alignments of the waves. Obstacles result in shadowing, or weakened signal strength, and the output of antennas varies by angle. All of this contributes to hard-to-describe signal reception conditions. Euclidean distance are therefore a poor descriptor of signal condition. It's though worth keeping in mind that simple abstractions have their place.

Fortunately, the additivity of interference (axiom (ii)) and thresholding of signal capture (axiom (iii)) of the SINR model generally hold up quite well in measurements [21], [16], [3], [20], [6]. It is the polynomial pathloss assumption that is primarily problematic.

III. BEYOND GEOMETRIC SINR

The lack of fidelity in the standard models, or the mismatch between actual and predicted signal properties, has motivated researchers to find ways to extend them or strengthen.

One simple but important observation is that "distances" need not refer to separation in physical Euclidean space. Distortions can be captured as modifications of the underlying metric space. Rather than specifying pathloss in terms of distance, the opposite is more natural: define "distance" in terms of the actual pathloss. Wireless nodes are generally equipped with carrier sense, which can be used to measure the pairwise signal pathloss. The model then moves from being *prescriptive* (of how reality must be like) to becoming *descriptive* of (how reality expresses itself).

Model 3 (Measured Pathloss (MEASSINR))**.** *Pathloss is given as a matrix $H = (h_{uv})$ (where $h_{uv} = P_{uv}/P_u$ is the pathloss between nodes u and v), obtained by measurements or projections. Other SINR axioms hold.*

An obvious way to generalize GEOSINR is to simply drop the assumption of polynomial pathloss.

Model 4 (Abstract SINR Model (ABSSINR))**.** *Axioms (ii) and (iii) hold, but signal can have arbitrary pathloss, not necessarily related to the distance.*

Unfortunately, this model is too pessimistic. It is easy to show that most interesting algorithmic problems become extremely hard, even to obtain weak approximations (e.g., [4]). Instead, we must search for restrictions that maintain reasonable levels of both fidelity and computability.

One approach is to assume restrictions on the pathloss matrix [2].

Property 1 (Metricity (ζ))**.** *The* metricity *ζ of the pathloss matrix $H = (h_{ij})$ is the smallest value such that, for every u, v, w,*

$$a_{uv}^{1/\zeta} + a_{vw}^{1/\zeta} \leq a_{uw}^{1/\zeta} \ .$$

Then, d forms a (quasi-)metric space, where $d(u, v) = a_{uv}^{1/\zeta}$ (note that symmetry need not hold).

We can then consider types of metric spaces of differing generality.

Option 2 (Metrics)**.** EUCLID*: Euclidean plane, with $\alpha > 2$;* METRIC*: General metric space;* FADING*: A doubling metric, where α is greater than the "doubling constant" of the metric. More precisely, metric is* fading *if, there is a constant γ, such that for every $t > 0$ and every set X nodes of mutual distance at least t, the total interference from X is at most proportional to the strength of a single transmission across distance t, i.e., $\sum_{x\in X} P_{xv} \leq \gamma \cdot t^\alpha$, for every node v.*

The fading metric [8] definition is meant to capture the essential property that spatial reuse is possible.

Relative interference (also known as *affectance*) represents the degree to which one transmission affects another one, i.e., proportional to the interference tolerance of the latter. First proposed [12], in its simplest form, the relative interference of node x (sender of link i) on link j is $a(x, j) = P_{s_i,r_j}/P_{s_j,r_j}$. A more refined version is $a(x, j) = \min(1, (1-\beta N/P_{s_j,r_j})P_{s_i,r_j}/P_{s_j,r_j})$ [5], [10], [14].

Link set definitions

Many scheduling problems involve a specific set L of communication links, where each link i is a sender-receiver pair (s_i, r_i). We can form a directed edge-weighted graph $G = G_L = (V, V \times V, w)$, where the nodes correspond to the links, edges form a complete graph, and the weight of edge (i, j) is $w(i, j) = a(s_i, j)$, the relative interference of i on j. Alternative, we can also form the undirected graph with weights $w(i, j) = a(s_i, j) + a(s_j, i)$.

Model 5 (Weighted Graphs on Links (WGL))**.** *Transmission on link i is successful if $\sum_{j\in S} w(j, i) \leq 1$, where S is the set of transmitting links.*

Such weighted graphs are simply another way of viewing the ABSSINR model, and are in general hard to compute with (e.g., schedule). The graphs induced by communication links have geometric structure that can be leveraged. They also satisfy a graph-theoretic property, bounded inductive independence, which allows for efficient solution of certain scheduling problems. As proposed by [14], [11], this property by itself could even be viewed as a model definition.

Property 2 (Inductive Independence($\rho(L)$))**.** *The* inductive independence *$\rho(L)$ of a set L of links is the smallest value t such that for each link $i \in L$ and each feasible subset $X \subseteq L$ of links at least as long as i, $\sum_{j\in X} w(i, j) \leq t$.*

This property can also be applied in the case of GENERAL power control, using the weight function $w(i, j)$ proportional to $c \cdot d(s_j, r_j)/\min(d(s_i, r_j), d(s_j, r_i))$ [13].

A generalization of the protocol model has recently been proposed in [9]. It is built on the observation that the protocol model can be overly conservative when links are of widely different lengths [17]. It has been shown that this generalized model can give a close graph-based approximation of the GEOSINR model, in that schedules in one model are within nearly constant factors of valid schedules in the other model [9].

We give here a variation that is node-based (rather than link-based); if a node belongs to multiple links, we can view it as having independent replicates.

Model 6 (Generalized Protocol Model (GPM))**.** *Let $f : \mathbb{R}^+ \to \mathbb{R}^+$ be a monotone non-decreasing function. Each node v has a given transmission radius r_v. The transmission of v to w is successful $d(v, w) \leq r_v$ and $d(v, u) > r_a f(r_b/r_a)$, for every other transmitting node u, where $r_a = \min(r_u, r_v)$ and $r_b = \max(r_u, r_v)$.*

IV. STOCHASTIC MODELS

Fading is the term used to denote the irregular or non-deterministic variability in wireless reception conditions. The most common approach to handling fading is to model it stochastically. The received signal strength is then a random variable whose expectation equals the geometric pathloss. There are several methods in use of differing levels of complexity; we shall focus on two of them.

The term fading actually encompasses two different aspects: the temporal variability, that reflects movement of sender/receiver or changes in the environment, and the non-temporal spatial differences. Experimental results indicate that in static environment experience little or no temporal fading.

Fading is frequently divided into two parts: *fast fading*, due to multiple signal transmission paths, and *slow fading*, that reflects more large-scale features of the environment, often referred to as the *shadowing* caused by intermediate objects [18]. The overall fading is then a combination of the two.

The most common model for fast fading, often assumed to be time varying, is Rayleigh fading.

Model 7 (Rayleigh Fading (RAY))**.** *The received strength of signal (or interference) from node u to node v is an exponentially distributed random variable $\overline{P_{uv}}$ with a mean of $P_u/d(u, v)^\alpha$. The other aspects of the SINR model hold.*

For large-scale non-temporal effects, log-normal shadowing appears to be the model of choice.

Model 8 (Log-Normal Shadowing (LNS))**.** *The received strength of signal (or interference) from node u to node v is a random variable $\overline{P_{uv}} = e^X$, where X is a Gaussian random variable with a mean μ and variance σ, such that $E[P_{uv}] = e^{\mu+\sigma^2/2} = P/d(u, v)^\alpha$. Thus, the logarithm of the signal strength has a Gaussian distribution.*

The hardest part of properly modeling these aspects stochastically is capturing the correlations between spatially close transmissions, that have been observed in measurements [1]. Most frequently, this aspect is omitted. The other issue is how independent the events are across time.

Stochastic modeling is an alternative to generalizing deterministic pathloss matrices to non-Euclidean distances. Arbitrary pathloss matrices allow for worst-case or "every-case" analysis, and in that sense are more general for capturing static aspects than stochastic modifications.

V. DISTRIBUTED ALGORITHMS

Distributed computing raises additional issues that do not concern centralized algorithms. It can be condensed to a question of knowledge: what do nodes know about the rest of the network, and what can they infer during the execution of the algorithm.

Option 3 (Carrier sense (CS))**.** Carrier sense *is the ability of nodes to detect activity on the channel by measurement. This can be used for* collision detection*, deciding whether the lack* *of reception meant that no message was received or multiple transmissions clashed.*

Option 4 (Location and Distances)**.** *The Euclidean* location *of a node may be available, if hard-coded or measured by GPS. A weaker property is to be able to measure* distance *from transmitting node, such as applying Carrier Sense to geometric pathloss.*

There are also various issues that arise in distributed computing in general, which become salient in wireless networks.

Option 5 (Synchronization)**.** *SYNC: Globally synchronized operation; LOCAL-SYNC: Local neighbors are synchronized, but not globally; ASYNC: No synchronization*

Option 6 (Wakeup)**.** SPONT*: All nodes spontaneously wake up at the same time.* NSPONT*: Nodes can wake other nodes up.* UNSTRUCTURED*: Nodes can wake up at different times*

The knowledge that nodes have about the network can crucially affect their operation.

Option 7 (Knowledge of Nodes)**.** *Relevant knowledge includes:*

- *Number of nodes, n;*
- *Length diversity (ratio between longest and shortest length), Λ;*
- *Degree (number of nodes within some fixed distance), Δ;*
- *Model parameters (e.g., α, β, N).*
- *Unique node identifiers*

In some cases, it may suffice to know only an approximation of the quantity or an upper bound.

VI. The Frontier

We cannot expect to be encyclopedic in our treatment of issues that affect algorithms for wireless networks. We outline below several key dimensions that are bound to be important for future models.

Unreliability: Perhaps the most challenging part of wireless networking is dealing with non-deterministic aspects. This can be divided into two categories: *unreliability*, in the form of intermittent connectivity or favorable signal propagation; and *time-variability*, the impact of mobility of users, movement in the environment, or other changes.

One graph-based model proposed to capture unreliability is *dual graphs* [15]. We propose the following variation that can be applied to any interference model.

Model 9 (Dual Graphs (DUAL))**.** *On top of a base model (PM or SINR), a subset of node pairs (or links) are unreliable. There are several options: NONE: No signal gets transmitted between the nodes; INT: Only interference gets propagated; STOCH: Signal is propagated, but decodability is probabilistic. ADV: Whether signal gets propagated is under adversarial control.*

The strength of the adversary can vary. Another option is how nodes can be aware of which links are unreliable.

Mobility is a also common issue, but also one that is hard to capture formally. Mostly, it is captured by empirical "mobility models".

Advanced PHY-Layer Models: Advances in technology at the physical layer are pushing the boundaries of what is possible in wireless communication, not just by gradual improvements, but by conceptually different modes of operation. The signal from multiple transmitters is synchronized to create either constructive or destructive alignments of the transmission waves. Additionally, the analog signal can be subtracted to reveal hidden signals. Some of the catchphrases include *beamforming*, *MIMO*, *interference alignment*, *interference cancellation*, and *analog network coding*. While these techniques have been studied for a quite some time from an information theory perspective, much is to be done on the algorithmic frontier.

VII. Discussion

This survey indicates the many potential avenues for enriching standard models with the aim of capturing more realism. Various aspects are yet to be fully exploited, particularly when it comes to unreliability and new PHY-layer technologies. We have also omitted various issues, relating to, e.g., data rates, energy use, and sleep and lifetime management.

If treated individually, the cornucopia of model variants and options could easily lead to a fragmentation and duplication of effort. Instead, it is essential that analytic research effort focus on theory building: deriving reusable results and techniques that are general enough to work for a range of problems under the widest possible range of models.

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