

# Computer Science and Logic A Match Made in Heaven

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Thanks to Moshe Vardi from whom I have drawn inspiration (read “stolen ideas”) for this presentation.

# Why This Talk Today?

The importance of logic for CS is perhaps not sufficiently appreciated.

## My message in a nutshell

Logic - used by mathematicians but not talked about,  
talked about by philosophers but not used.

Talked about, used and revitalized by computer scientists!

Roadmap of the talk:

- 1 Background
- 2 Logic and CS: Past and present
- 3 Logic from CS
- 4 Conclusion

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# Mathematics and the Natural Sciences

E.P. Wigner (1960): [On the Unreasonable Effectiveness of Mathematics in the Natural Sciences](#).

- “The book of nature is written in the language of mathematics.” (Galileo)
- Numerous examples of the effectiveness of mathematics in the physical sciences.

The empirical law of epistemology

The mathematical formulation of the laws of nature is both appropriate and accurate. **Mathematics is the correct language for formulating the laws of nature.**

Question: What is the correct language for computer science?

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# Mathematical Logic

Mathematical logic was developed in an attempt to confront the crisis in the foundations of mathematics at the turn of the 20th century.

## Hilbert's Program (1900–1928)

Formalize mathematics and establish that it is

- **consistent**,
- **complete** and
- **decidable**.

Hilbert firmly believed that these goals can be achieved.

# The Demise of Hilbert's Program

## K. Gödel (1931–1933):

- Incompleteness of ordinary arithmetic.
- Impossibility of proving consistency within “reasonably rich formalisms”.

## A. Church and A. Turing (1936–1937): Undecidability of first-order logic

- The set of valid first-order sentences is **not** decidable.
- The set of all first-order sentences that true in arithmetic is **not** recursively enumerable.

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# Where is Logic Today?

Google, 2 April 2009 at 21:37 GMT:

- Logic and Computer Science: roughly 13,700,000 hits
- Logic and Mathematics: roughly 3,310,000 hits
- Logic and Philosophy: roughly 6,250,000 hits

# Logic in Computer Science

During the last 30 years, there has been extensive and growing interaction between logic and computer science.

Concepts and methods of logic play a central role in CS: **logic is the calculus of CS.**

Logic has been much more effective in CS than it has been in mathematics.

M. Davis (1988)

When I was a student, even the topologists regarded mathematical logicians as living in outer space. Today, the connections between logic and computers are a matter of engineering practice at every level of computer organization.

# Why?

## Alan Turing

I expect that digital computing machines will eventually stimulate a considerable interest in symbolic logic. . . The language in which one communicates with these machines . . . forms a sort of symbolic logic.

Logic provides:

- formalisms to describe mathematical structures
- languages to describe properties of mathematical structures
- languages to describe dynamic processes
- a clear distinction between syntax and semantics

**Michael Jackson:** “Description is our business”

## And Logic Begat CS!

When dinosaurs roamed the earth, there was logic:

- Outcome of Hilbert's program: formalization of algorithms
- Stored-program computer: universal Turing machine
- Functional programming: the  $\lambda$ -calculus
- Recursive-function theory: precursor of complexity theory
- Automata theory: finite automata

Yes, but that was then. What about now?

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## Impact of Logic in CS

Logic has been effective in several different areas of CS, including:

- Computer-aided verification
- Database systems
- Computational complexity
- Programming languages
- Computer security
- Artificial intelligence
- Digital design
- Logic programming (of course!)
- Software engineering
- ... (add you own favourite!)

## Exhibit 1: Computer-aided verification

Model Checking = Graphs + Logic + Algorithms.

- 1 Graphs describe the behaviour of the design space to be analyzed. (Nodes = States, Edges = State Transitions)
- 2 Logic is used to describe (un)desirable properties of the design. (Every sent message will eventually be delivered.)
- 3 Algorithms are used for a systematic exploration of the design state space.

Impact: Widely used in the HW industry and spreading in the SW industry.



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## Exhibit 2: Logic and databases

A multi-billion dollar success!

- First-order logic lies at the core of modern database systems (SQL, QBE)
- Efficient query evaluation is based on **relational algebra** (aka **cylindrical algebra** (Alfred Tarski))

Semistructured data:

- XML: trees
- Data Type Definition: Tree automata
- XML queries: tree transducers

## Exhibit 3: Logic and complexity

Complexity:

- **Computational complexity:** amount of resources, such as time or space, required by a machine that solves a problem.
- **Descriptive complexity:** syntactic complexity of describing a problem in some logical formalism.

An amazing connection: The two are intimately related!

Machine independent	Machine based
FO(LFP)	PTIME
SO $\exists$	NP
SO(TC)	PSPACE

## Exhibit 4: Logic and programming languages

Logic has been the primary tool in the systematic development of programming languages. In particular,

- formal semantics
- type theory
- rewriting systems
- ... (add you own favourite!)

Logic is not only used to design languages and compilers, but is also applied directly to “real” software artifacts.

## Exhibit 5: Logic and security

Areas of applicability: numerous!

- Access control policies
- Trust management
- Cryptographic protocol analysis
- ... (add you own favourite!)

Means of application: numerous!

- Authentication logics
- Temporal and epistemic logics
- Linear logic
- Model checking
- Theorem proving

# Logic from CS

CS problems invigorate logic research.

- Unlike traditional mathematical logic, we focus on **the study of logics over finite structures** (e.g., databases): birth of **finite-model theory**.
- CS studies powerful extensions of first-order logic: e.g., **fixed-point logics in concurrency theory and databases**
- Emphasis on the connection between logics and automata.
- Major role for modal and temporal logics.
- New logics have to be invented, e.g., authentication logics in security.
- New notions of inference, e.g., **non-monotonic inference in AI**.
- Focus on logics of uncertainty, e.g., **in decision support systems**.

# Logic in CS

- Logic is mainly used as a formalism for describing objects and specifying their properties.
- Logic in CS is an **applied science** combining foundational research with applications. (See, e.g., the work on SAT solvers.)
- Logic (in CS) is an **experimental science!**

## Take-home Message

- Logic is the most applied branch of mathematics today, thanks to CS.
- The rules of logic, not those of Nature, put limits and frame the work of a computer scientist. Hence, CS practitioners must know some logic.
- IMHO, logic ought to be integrated in the CS curriculum.



## Take-home Quotation

Arnold Schwarzenegger in “Twins”

You have **no** respect for logic;  
I have **no** respect for people  
who have **no** respect for logic.

Arnold proceeds to beat up the thug.

Thank you!

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